Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 1, (July 2011), pp. 69- 80
ISSN 2093–9310
http://www.afmi.or.kr

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Soft rough sets applied to multicriteria group decision making

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Received 31 January 2011; Accepted 26 February 2011

Abstract. Molodtsov's soft sets have been applied by several authors to the study of decision making under uncertainty. In this study, we aim to initiate the application of soft rough approximations in multicriteria group decision making problems. A soft rough set based multicriteria group decision making scheme is presented, which refines the primary evaluation of the whole expert group and enables us to select the optimal object in a more reliable manner. The proposed scheme is illustrated by a concrete example regarding the house purchase problem.

2010 AMS Classification: 03E72, 90B50, 06D72

Keywords: Soft set, Soft rough set, Fuzzy set, Fuzzy soft set, Decision making.

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1. Introduction

The mathematical modelling of vagueness and uncertainty has become an increasingly important issue in diverse research areas. Various formal settings, including but not restricted to probability theory, fuzzy sets, rough sets and interval mathematics, have been proposed based on different intuitions. Although these theories can successfully be used to extract useful information hidden in uncertain data, each of them has its inherent difficulties. According to Molodtsov [17], one reason may be due to the inadequacy of the parametrization tool of the theory. In 1999, Molodtsov [17] initiated the concept of *soft sets* as a general mathematical approach for dealing with uncertainty, which is free from the difficulties affecting the existing methods. Recently there has been a rapid growth of interest in soft set theory and its applications. Many efforts have been devoted to further generalizations and extensions of Molodtsov's soft sets. Maji et al. [16] defined *fuzzy soft sets*, combining soft sets with fuzzy sets. This line of exploration was further investigated by several researchers [14, 20, 21]. Maji et al. [13] reported a detailed theoretical study on soft sets, with emphasis on the algebraic operations. Jiang et al. [7] extended soft sets with description logics. Aktaş and Çağman [2] initiated the notion of soft groups, extending fuzzy groups. Jun et al. discussed the applications of soft sets to the study of BCK/BCI-algebras [9, 10, 11, 12]. Feng et al. investigated the relationships among soft sets, rough sets and fuzzy sets, obtaining three types of hybrid models: rough soft sets, soft rough sets, and soft-rough fuzzy sets [3, 4]. The application of soft sets in decision making problems was initiated in [15]. To address fuzzy soft set based decision making problems, Roy and Maji [19] presented a novel method of object recognition from an imprecise multi-observer data. Using level soft sets, Feng et al. [5] proposed an adjustable approach to (weighted) fuzzy soft set based decision making. This approach was further investigated in [6, 8].

Although Molodtsov's soft sets have been applied by several authors to the study of decision making under uncertainty, it seems that soft set based group decision making has not been discussed yet in the literature. Thus the present study can be seen as a first attempt toward the possible application of soft rough approximations in multicriteria group decision making under uncertainty. In the proposed scheme, each expert simply gives an initial set consisting of the preferable alternatives in the corresponding expert's point of view. The primary evaluation results of the expert group will be stored in the evaluation soft set and then approximated in the original description soft set using soft rough approximations. Finally, all the obtain data of the whole expert group can be synthesized into a fuzzy soft set and all the alternatives will be ranked according to their weighted evaluation values. It should be noted that the use of soft rough sets could, to some extent, automatically reduce the noise factor caused by the subjective nature of the expert's evaluation. A concrete example concerning house selection is presented to illustrate how to use our method in practical applications.

2. Soft set theory

A soft set is a family of crisp sets (in a given universe) organized as a whole using some parameters (usually mean attributes, characteristics, or properties). More formally, let U be a universe of discourse and let E be the universal set of parameters related to objects in U. Here we only consider the case where both U and E are nonempty finite sets. Let $\mathscr{P}(U)$ denote the power set of U. Following the definition in [2], the concept of soft sets is defined as follows.

Definition 2.1 ([2])**.** A pair $S = (F, A)$ is called a *soft set* over U, where $A \subseteq E$ and $F\colon A \to \mathscr{P}(U)$ is a set-valued mapping, called the *approximate function* of S.

In other words, a soft set is a parameterized family of (crisp) subsets of the universe of discourse. For $\epsilon \in A$, $F(\epsilon)$ may be interpreted as the set of *ϵ-approximate elements*, and called an *ϵ-approximation.* It is worth noting that $F(\epsilon)$ may be arbitrary: some of them may be empty, and some may have nonempty intersection [17]. To illustrate this idea, we shall consider the following house purchase problem.

Example 2.2. Assume that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ is a universe consisting of five houses as possible alternatives, and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ is a set of parameters considered by the decision makers, where e_1, e_2, e_3 and e_4 represent the parameters"beautiful", "modern", "cheap" and "in the green surroundings", respectively. Now, we consider a soft set $\mathfrak{S} = (F, A)$ which describes the "attractiveness of the houses" that Mr. X is going to buy. In this case, to define the soft set (F, A) means to point out beautiful houses, modern houses and so on. Consider the mapping F given by "houses$(\cdot)$", where $(\cdot)$ is to be filled in by one of the parameters $e_i \in A$. For instance, $F(e_1)$ means "houses(beautiful)", and its functional value is the set consisting of all the beautiful houses in U. Let $F(e_1) = \{h_5, h_7\}$, $F(e_2) = \{h_1, h_4, h_6, h_7\}$, $F(e_3) = \{h_1, h_3\}$ and $F(e_4) = \{h_2, h_4, h_5\}$. Tabular representation of the soft set $\mathfrak{S}$ is given by Table 1.

Table 1. Tabular representation of the soft set $\mathfrak{S}$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
e_1	0	0	0	0	1	0	1
e_2	1	0	0	1	0	1	1
e_3	1	0	1	0	0	0	0
e_4	0	1	0	1	1	0	0

Definition 2.3 ([1]). Let (F, A) and (G, B) be two soft sets over U. Then (G, B) is called a *soft subset* of (F, A), denoted by $(G, B) \subseteq (F, A)$, if $B \subseteq A$ and $G(b) \subseteq F(b)$ for all $b \in B$.

The theory of *fuzzy sets*, initiated by Zadeh [22] in 1965, provides a useful framework for modelling and manipulating vague concepts. Recall that a fuzzy subset A of U is defined by its *membership function* $\mu_A : U \to [0, 1]$. For $x \in U$, the membership value $\mu_A(x)$ can be interpreted as the degree to which $x \in U$ belongs to the fuzzy set A. Conventionally, we identify a fuzzy set A with its membership function μ_A.

Let $\mathscr{F}(U)$ denote the set of all fuzzy subsets of U. By combining fuzzy sets with soft sets, Maji et al. [16] initiated the following hybrid model.

Definition 2.4 ([16]). A pair $\mathfrak{S} = (\widetilde{F}, A)$ is called a *fuzzy soft set* over U, where $A \subseteq E$ and $\widetilde{F} : A \to \mathscr{F}(U)$ is the approximate function of $\mathfrak{S}$.

Clearly, fuzzy soft sets extend classical soft sets by substituting fuzzy subsets for just crisp subsets of the universe. Note also that fuzzy sets can be viewed as fuzzy soft sets with a single parameter.

3. Soft rough approximations

In this section, we introduce soft rough approximations and soft rough sets, initiated by the author in [3]. For more details on this topic, we refer the interested reader to [4]. All proofs omitted can be found there.

Definition 3.1. Let $S = (F, A)$ be a soft set over U. Then the pair $P = (U, S)$ is called a *soft approximation space*. Based on the soft approximation space P, we define the following two operations

$$\begin{aligned} \underline{apr}_P(X) &= \{u \in U : \exists a \in A, [u \in F(a) \subseteq X]\}, \\ \overline{apr}_P(X) &= \{u \in U : \exists a \in A, [u \in F(a), F(a) \cap X \neq \varnothing]\}, \end{aligned}$$

assigning to every subset $X \subseteq U$ two sets $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$, which are called the *soft P-lower approximation* and the *soft P-upper approximation* of X, respectively. In general, we refer to $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ as *soft rough approximations* of X with respect to P. Moreover, the sets

$$\begin{aligned} Pos_P(X) &= \underline{apr}_P(X), \\ Neg_P(X) &= U - \overline{apr}_P(X) \\ Bnd_P(X) &= \overline{apr}_P(X) - \underline{apr}_P(X) \end{aligned}$$

are called the *soft P-positive region*, the *soft P-negative region* and the *soft P-boundary region* of X, respectively. If $\underline{apr}_P(X) = \overline{apr}_P(X)$, X is said to be *soft P-definable*; otherwise X is called a *soft P-rough set*.

By Definition 3.1, we immediately have that $X \subseteq U$ is a soft P-definable set if the soft P-boundary region $Bnd_P(X)$ of X is empty. Also, it is clear that $\underline{apr}_P(X) \subseteq X$ and $\underline{apr}_P(X) \subseteq \overline{apr}_P(X)$ for all $X \subseteq U$. Nevertheless, it is worth noticing that $X \subseteq \overline{apr}_P(X)$ does not hold in general.

The following result is easily obtained from the definition of soft rough approximations.

Proposition 3.2. *Let $S = (F, A)$ be a soft set over U and $P = (U, S)$ a soft approximation space. Then we have*

$$\underline{apr}_P(X) = \bigcup_{a \in A} \{F(a) : F(a) \subseteq X\}$$

and

$$\overline{apr}_P(X) = \bigcup_{a \in A} \{F(a) : F(a) \cap X \neq \varnothing\}$$

for all $X \subseteq U$.

Suppose that $S = (F, A)$ is a soft set over U and $P = (U, S)$ is the corresponding soft approximation space. One can verify that soft rough approximations satisfy the following properties:

(1) $\underline{apr}_P(\varnothing) = \overline{apr}_P(\varnothing) = \varnothing$;

(2) $\underline{apr}_P(U) = \overline{apr}_P(U) = \bigcup_{a \in A} f(a)$;

(3) $\underline{apr}_P(X \cap Y) \subseteq \underline{apr}_P(X) \cap \underline{apr}_P(Y)$;

(4) $\underline{apr}_P(X \cup Y) \supseteq \underline{apr}_P(X) \cup \underline{apr}_P(Y)$;

(5) $\overline{apr}_P(X \cup Y) = \overline{apr}_P(X) \cup \overline{apr}_P(Y)$;

(6) $\overline{apr}_P(X \cap Y) \subseteq \overline{apr}_P(X) \cap \overline{apr}_P(Y)$;

(7) $X \subseteq Y \Rightarrow \underline{apr}_P(X) \subseteq \underline{apr}_P(Y)$;

(8) $X \subseteq Y \Rightarrow \overline{apr}_P(X) \subseteq \overline{apr}_P(Y)$;

(9) $\underline{apr}_P(\overline{apr}_P(X)) = \overline{apr}_P(X)$;

(10) $\overline{apr}_P(\underline{apr}_P(X)) \supseteq \underline{apr}_P(X)$;

(11) $\underline{apr}_P(\underline{apr}_P(X)) = \underline{apr}_P(X)$;

(12) $\overline{apr}_P(\overline{apr}_P(X)) \supseteq \overline{apr}_P(X)$.

Definition 3.3. Let $S = (F, A)$ be a soft set over U. If $\bigcup_{a \in A} F(a) = U$, then S is said to be a *full soft set*.

Definition 3.4. A soft set $S = (F, A)$ over U is call a *partition soft set* if $\{F(a) : a \in A\}$ forms a partition of U.

To show the relationship between soft rough sets and Pawlak's rough sets, we first observe that soft sets and binary relations are closely related [3, 4].

Theorem 3.5. *Let $S = (F, A)$ be a soft set over U. Then S induces a binary relation $\rho_S \subseteq A \times U$, which is defined by*

$$(x, y) \in \rho_S \Leftrightarrow y \in F(x)$$

for all $x \in A$ and $y \in U$.

Conversely, let ρ be a binary relation from A to U. Define a set-valued mapping $F_\rho : A \to \mathscr{P}(U)$ by

$$F_\rho(x) = \{y \in U : (x, y) \in \rho\}$$

for all $x \in A$. Then $S_\rho = (F_\rho, A)$ is a soft set over U. Moreover, we have that $S_{\rho_S} = S$ and $\rho_{S_\rho} = \rho$.

The following results show that Pawlak's rough set model can be viewed as a special case of soft rough sets [4].

Theorem 3.6. *Let R be an equivalence relation on U, $S_R = (F_R, U)$ the canonical soft set of R and $P = (U, S_R)$ a soft approximation space. Then for all $X \subseteq U$,*

$$R_*X = \underline{apr}_P(X) \quad and \quad R^*X = \overline{apr}_P(X),$$

where R_*X and R^*X are the Pawlak rough approximations of X. Thus in this case, $X \subseteq U$ is a (Pawlak) rough set if and only if X is a soft P-rough set.

Theorem 3.7. *Let $S = (F, A)$ be a partition soft set over U and $P = (U, S)$ a soft approximation space. Define an equivalence relation R on U by*

$$(x, y) \in R \Leftrightarrow \exists a \in A, \{x, y\} \subseteq F(a)$$

for all $x, y \in U$. Then, for all $X \subseteq U$,

$$R_*X = \underline{apr}_P(X) \quad and \quad R^*X = \overline{apr}_P(X).$$

4. Multicriteria group decision making using soft rough sets

One of the most important applications of rough sets is to induce useful decision or classification rules [18]. Both soft sets and fuzzy soft sets have been also applied by many authors to solving decision making problems [5, 6, 15, 19]. In this section, we simply illustrate the use of soft sets, soft rough sets and related notions in object evaluation and group decision making.

Let $U = \{o_1, o_2, \cdots, o_l\}$ be a set of objects and E a set of related parameters. Let $A = \{e_1, e_2, \cdots, e_m\} \subseteq E$ and $S = (F, A)$ be an original *description soft set* over U. For real-life applications, we can always require that S is a full soft set over U.

Assume that we have an expert group $G = \{T_1, T_2, \cdots, T_n\}$ consisting of n specialists to evaluate the objects in U. Each specialist need to examine all the objects in U and will be requested to only point out "the optimal alternatives" as his/her evaluation result. Hence each specialist's primary evaluation result is a subset of U. For simplicity, we assume that the evaluations of these specialists in G are of the same importance. Let X_i denote the primary evaluation result of the specialist T_i. It is easy to see that the primary evaluation result of the whole expert group G can be represented as an *evaluation soft set* $S_1 = (\vartheta, G)$ over U, where $\vartheta : G \to \mathscr{P}(U)$ is given by $\vartheta(T_i) = X_i$ $(i = 1, 2, \cdots, n)$.

Clearly, the soft set $S_1 = (\vartheta, G)$ only provides us with an initial data set of evaluation. We may expect to gain much more useful information with the help of soft rough approximations. Specifically, we can consider the soft rough approximations of the specialist T_i's primary evaluation result X_i with respect to the soft approximation space $P = (U, S)$. The soft lower approximation $\underline{apr}_P(X_i)$ can be interpreted as the set consisting of the objects which are certainly the optimum candidates according to specialist T_i's evaluation. For instance, if $o_1 \in \underline{apr}_P(X_2)$ we can say that the specialist T_2 thinks with high confidence that o_1 is an optimal alternative. This interpretation follows from the fact that $o_1 \in \underline{apr}_P(X_2)$ means that there exists at least one parameter $\epsilon \in A$ such that $F(\epsilon) \subseteq X_2$, whence we deduce the parameter ϵ must be a very strong criterion for the specialist T_2 to make the decision. Similarly, the soft upper approximation $\overline{apr}_P(X_i)$ can be interpreted as the set consisting of the objects which are possibly the optimum candidates according to specialist T_i's evaluation.

Using soft rough approximations, we finally obtain two other soft sets $\underline{S_1} = (\underline{\vartheta}, G)$ and $\overline{S_1} = (\overline{\vartheta}, G)$ over U, where $\underline{\vartheta} : G \to \mathscr{P}(U)$ is given by $\underline{\vartheta}(T_i) = \underline{apr}_P(X_i)$ and $\overline{\vartheta} : G \to \mathscr{P}(U)$ is given by $\overline{\vartheta}(T_i) = \overline{apr}_P(X_i)$ for $i = 1, 2, \cdots, n$. As mentioned above, the soft set $\underline{S_1}$ can be seen as the evaluation result of the whole expert group G with high confidence, while $\overline{S_1}$ represents the evaluation result of the whole expert group G with low confidence. Moreover, the primary evaluation namely the soft set S_1 may be also interpreted as the evaluation result of the whole group with middle confidence. It is easy to see that $\underline{S_1} \subseteq S_1 \subseteq \overline{S_1}$, provided that $S = (F, A)$ is a full soft set over U.

It is worth noting that the evaluation result of the whole expert group G could be also formulated in terms of fuzzy sets. For $X \subseteq U$, the characteristic function of X is denoted by C_X. Based on the soft set $S_1 = (\vartheta, G)$, we can define a fuzzy set

μ_{S_1} in U by

$$\mu_{S_1} : U \to [0,1], \quad o_k \mapsto \mu_{S_1}(o_k) = (1/n) \sum_{j=1}^{n} C_{\vartheta(T_j)}(o_k),$$

where $\vartheta(T_j) = X_j$ and $k = 1, 2, \cdots, l$. In a similar way, we obtain two other fuzzy sets $\mu_{\underline{S_1}}$ and $\mu_{\overline{S_1}}$ in U, which are respectively given by

$$\mu_{\underline{S_1}} : U \to [0,1], \quad o_k \mapsto \mu_{\underline{S_1}}(o_k) = (1/n) \sum_{j=1}^{n} C_{\underline{\vartheta}(T_j)}(o_k),$$

and

$$\mu_{\overline{S_1}} : U \to [0,1], \quad o_k \mapsto \mu_{\overline{S_1}}(o_k) = (1/n) \sum_{j=1}^{n} C_{\overline{\vartheta}(T_j)}(o_k),$$

where $\underline{\vartheta}(T_j) = \underline{apr}_P(X_j)$, $\overline{\vartheta}(T_j) = \overline{apr}_P(X_j)$ and $k = 1, 2, \cdots, l$.

From $\underline{S_1} \subseteq S_1 \subseteq \overline{S_1}$, it is easy to see that $\mu_{\underline{S_1}} \leq \mu_{S_1} \leq \mu_{\overline{S_1}}$. These fuzzy sets $\mu_{\underline{S_1}}$, μ_{S_1} and $\mu_{\overline{S_1}}$ can naturally be interpreted as some vague concepts like "optimal alternatives with high confidence", "optimal alternatives with middle confidence" and "optimal alternatives with low confidence", respectively.

Now, we can use the concept of fuzzy soft sets to combine the above "soft" or "fuzzy" evaluation results. Let $C = \{L, M, H\}$ be a set of parameters, where L, M and H represent "low confidence", "middle confidence" and "high confidence", respectively. Then we can define a fuzzy soft set $R = (\alpha, C)$ over U, where $\alpha : C \to \mathscr{F}(U)$ is give by $\alpha(L) = \mu_{\overline{S_1}}$, $\alpha(M) = \mu_{S_1}$ and $\alpha(H) = \mu_{\underline{S_1}}$. Given a weighting vector $W = (w_L, w_M, w_H)$ such that $w_L + w_M + w_H = 1$, we define

$$v(o_k) = w_L * \alpha(L)(o_k) + w_M * \alpha(M)(o_k) + w_H * \alpha(H)(o_k),$$

which is called the *weighted evaluation value* of the alternative $o_k \in U$. Finally we can select the object o_p such that $v(o_p) = \max\{v(o_k) : k = 1, 2, \cdots, l\}$, as the most preferred alternative.

The soft rough set based decision making method can be summarized as follows:

Step 1 Input the original description soft set $S = (F, A)$.

Step 2 Construct the evaluation soft set $S_1 = (\vartheta, G)$ using the primary evaluation results of the expert group G.

Step 3 Compute soft rough approximations and then obtain the soft sets $\underline{S_1} = (\underline{\vartheta}, G)$ and $\overline{S_1} = (\overline{\vartheta}, G)$.

Step 4 Compute the corresponding fuzzy sets μ_{S_1}, $\mu_{\underline{S_1}}$ and $\mu_{\overline{S_1}}$ of the soft sets $S_1 = (\vartheta, G)$, $\underline{S_1} = (\underline{\vartheta}, G)$ and $\overline{S_1} = (\overline{\vartheta}, G)$.

Step 5 Construct the fuzzy soft set $R = (\alpha, C)$ using the fuzzy sets μ_{S_1}, $\mu_{\underline{S_1}}$ and $\mu_{\overline{S_1}}$.

Step 6 Input the weighting vector W and compute the weighted evaluation values $v(o_i)$ of each alternative $o_i \in U$. Then rank all the alternatives according to their weighted evaluation values; one can select any of the objects with the largest weighted evaluation value as the most preferred alternative.

5. An illustrative example with comparative analysis

For a concrete example of the above idea, we revisit the house purchase problem in Example 2.2. So let us consider the soft set $\mathfrak{S} = (F, A)$ (see also Table 1 for its tabular representation) in Example 2.2, which describes the "attractiveness of the houses" that Mr. X is going to buy. Assume that we have an expert group $G = \{T_1, T_2, T_3\}$ of three specialists (say Mr. X and his family members) to evaluate the houses in U. Let X_i denote the primary evaluation result of the specialist T_i for $i = 1, 2, 3$. As was stated above, the primary evaluation of the whole expert group can be represented by the evaluation soft set $\mathfrak{S}_1 = (\vartheta, G)$ over U, whose tabular representation is given by Table 2.

Table 2. Tabular representation of the soft set $\mathfrak{S}_1$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
T_1	0	0	0	1	1	0	1
T_2	1	0	1	0	0	0	1
T_3	0	1	1	0	1	0	0

From the tabular representation of the soft set $\mathfrak{S}_1$, we know the primary evaluation results of the specialists are $X_1 = \vartheta(T_1) = \{h_4, h_5, h_7\}$, $X_2 = \vartheta(T_2) = \{h_1, h_3, h_7\}$ and $X_3 = \vartheta(T_3) = \{h_2, h_3, h_5\}$.

Now, we show how to use soft rough sets to support this group decision making process. Let us choose $P = (U, \mathfrak{S})$ as the soft approximation space. Then by calculations using Proposition 3.2, we have

$$\begin{aligned}
\underline{\vartheta}(T_1) &= \underline{apr}_P(X_1) = F(e_1) = \{h_5, h_7\}, \\
\underline{\vartheta}(T_2) &= \underline{apr}_P(X_2) = F(e_3) = \{h_1, h_3\}, \\
\underline{\vartheta}(T_3) &= \underline{apr}_P(X_3) = \varnothing,
\end{aligned}$$

and

$$\begin{aligned}
\overline{\vartheta}(T_1) &= \overline{apr}_P(X_1) = F(e_1) \cup F(e_2) \cup F(e_4) = U - \{h_3\}, \\
\overline{\vartheta}(T_2) &= \overline{apr}_P(X_2) = F(e_1) \cup F(e_2) \cup F(e_3) = U - \{h_2\}, \\
\overline{\vartheta}(T_3) &= \overline{apr}_P(X_3) = F(e_1) \cup F(e_3) \cup F(e_4) = U - \{h_6\}.
\end{aligned}$$

Based on the above soft rough approximations, we get two soft sets $\underline{\mathfrak{S}_1} = (\underline{\vartheta}, G)$ and $\overline{\mathfrak{S}_1} = (\overline{\vartheta}, G)$ over U, where $\underline{\vartheta}(T_i) = \underline{apr}_P(X_i)$ and $\overline{\vartheta}(T_i) = \overline{apr}_P(X_i)$ for $i = 1, 2, 3$. Tabular representations of these two soft sets are given by Table 3 and Table 4, respectively.

Table 3. Tabular representation of the soft set $\underline{\mathfrak{S}_1}$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
T_1	0	0	0	0	1	0	1
T_2	1	0	1	0	0	0	0
T_3	0	0	0	0	0	0	0

Table 4. Tabular representation of the soft set $\overline{\mathfrak{S}_1}$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
T_1	1	1	0	1	1	1	1
T_2	1	0	1	1	1	1	1
T_3	1	1	1	1	1	0	1

Now we can define the fuzzy sets $\mu_{\underline{\mathfrak{S}_1}}$, $\mu_{\mathfrak{S}_1}$ and $\mu_{\overline{\mathfrak{S}_1}}$ as follows:

$$\begin{aligned}
\mu_{\underline{\mathfrak{S}_1}}(h_k) &= (1/3)\sum_{j=1}^{3} C_{\underline{\vartheta}(T_j)}(h_k),\\
\mu_{\mathfrak{S}_1}(h_k) &= (1/3)\sum_{j=1}^{3} C_{\vartheta(T_j)}(h_k),\\
\mu_{\overline{\mathfrak{S}_1}}(h_k) &= (1/3)\sum_{j=1}^{3} C_{\overline{\vartheta}(T_j)}(h_k),
\end{aligned}$$

where $k = 1, 2, \cdots, 7$. By calculations using the above formulae, we have

$$\begin{aligned}
\mu_{\underline{\mathfrak{S}_1}} &= \{(h_1, 1/3), (h_2, 0), (h_3, 1/3), (h_4, 0), (h_5, 1/3), (h_6, 0), (h_7, 1/3)\},\\
\mu_{\mathfrak{S}_1} &= \{(h_1, 1/3), (h_2, 1/3), (h_3, 2/3), (h_4, 1/3), (h_5, 2/3), (h_6, 0), (h_7, 2/3)\},\\
\mu_{\overline{\mathfrak{S}_1}} &= \{(h_1, 1), (h_2, 2/3), (h_3, 2/3), (h_4, 1), (h_5, 1), (h_6, 2/3), (h_7, 1)\}.
\end{aligned}$$

Let $C = \{L, M, H\}$ be a set of parameters, where L, M and H represent "low confidence", "middle confidence" and "high confidence", respectively. Then we obtain a fuzzy soft set $R = (\alpha, C)$ over U by setting $\alpha(L) = \mu_{\overline{\mathfrak{S}_1}}$, $\alpha(M) = \mu_{\mathfrak{S}_1}$ and $\alpha(H) = \mu_{\underline{\mathfrak{S}_1}}$. Assume that the weighting vector $W = (0.25, 0.5, 0.25)$. We calculate the weighted evaluation value

$$v(h_k) = 0.25 * \alpha(L)(h_k) + 0.5 * \alpha(M)(h_k) + 0.25 * \alpha(H)(h_k),$$

of each house $h_k \in U$ $(k = 1, 2, \cdots, 7)$. Tabular representation of the fuzzy soft set $R = (\alpha, C)$ with evaluation values is given by Table 5. From the table, we can find the ranking of all the alternatives with respect to their weighted evaluation values:

$$h_5 \approx h_7 \succ h_3 \succ h_1 \succ h_4 \succ h_2 \succ h_6.$$

Hence h_5 or h_7 should be the most preferred houses for Mr. X to consider for purchase.

Table 5. Fuzzy soft set $R = (\alpha, C)$ with weighted evaluation values

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
L	1	2/3	2/3	1	1	2/3	1
M	1/3	1/3	2/3	1/3	2/3	0	2/3
H	1/3	0	1/3	0	1/3	0	1/3
$v(\cdot)$	0.5	0.33	0.58	0.42	0.67	0.17	0.67

At the end of our discussion, we give a comparative analysis of several decision making methods and point out some of the advantages of the approach based on soft rough sets. First, one can observe that there exists conflict between the members of the expert group since $\vartheta(T_1) \cap \vartheta(T_2) \cap \vartheta(T_3) = \varnothing$. Thus we cannot take out the best alternative directly from the primary evaluation results. Also it is easy to find that the ranking of the alternatives based on the primary evaluation results should be as follows:

$$h_3 \approx h_5 \approx h_7 \succ h_1 \approx h_2 \approx h_4 \succ h_6,$$

which gives little information to support the decision making process.

It is also worth noting that this decision making problem can hardly be solved using the traditional soft set based method initiated by Maji et al. [15]. Specifically, using Maji's decision scheme the final decision will be made solo by just one decision maker according to the choice values calculated from the original description soft set. By simple calculation, one can obtain that the choice value of the houses are $c_1 = c_4 = c_5 = c_7 = 2$ and $c_2 = c_3 = c_6 = 1$; hence the ranking of the alternatives based on choice values will be the following:

$$h_1 \approx h_4 \approx h_5 \approx h_7 \succ h_2 \approx h_3 \approx h_6.$$

This means that the traditional decision making method is almost useless to the cases of our example. However, in the soft rough set based method proposed above, the final optimal decision is not only based on the original description soft set $\mathfrak{S} = (F, A)$, but also relevant to the evaluation soft set $\mathfrak{S}_1 = (\vartheta, G)$. This is more reasonable since decision making is closely related to object evaluation in many real-life applications. In addition, the evaluation soft set contains the evaluation results of a group of experts. This is also very meaningful since in the real world, many important decision is made by an expert group, instead of only a single decision maker. For instance, Mr. X usually need to consult his family members when he decide to buy a new house.

Furthermore, we cannot make use of Pawlak's rough set model in this setting as well. Actually, it is clear that the indiscernibility relation $I(A)$ induced by the parameter set A is given by

$$(x, y) \in I(A) \Leftrightarrow C_{F(e_i)}(x) = C_{F(e_i)}(y), \forall e_i \in A,$$

where $x, y \in U$, and $C_{F(e_i)}$ denotes the characteristic function of $F(e_i)$. For the soft set $\mathfrak{S} = (F, A)$ in Example 2.2, we obtain that $I(A)$ is the identity relation $\Delta = \{(x, x) \mid x \in U\}$. It follows that the quotient set $U/I(A)$ is $\{[x]_{I(A)} \mid x \in U\}$, and so every subset $X \subseteq U$ is definable. This means that with Pawlak's rough approximations, we cannot further extract useful information from the collected data of the problem. One may suggest to use a subset of A instead, but this is not the case since A just contains all the parameters which are considered by the decision makers in this problem, and thus any of the parameters in A should not be ignored.

In the end, it should be noted that the use of the soft rough technique in our new proposal also refines the primary evaluation results of the whole expert group and thus enables us to select the optimal object in a more reliable manner. Specifically, the soft upper approximation can be used to add the optimal objects possibly neglected by some experts in the primary evaluation, while the soft lower approximation can be used to remove the objects that are improperly selected as the optimal objects by some experts in the primary evaluation. Therefore, while the subjective aspect of decision making is considered and described by the evaluation soft set in our new proposal, the use of soft rough sets could, to some extent, automatically reduce the errors caused by the subjective nature of the evaluation given by an expert group in some decision making problems.

6. Conclusions

We have proposed a soft rough set based scheme for supporting multicriteria group decision making, illustrated by a concrete example regarding the house purchase problem. To extend this work, one can consider to apply soft rough approximations of fuzzy sets to multicriteria group decision making problems.

Acknowledgements. The author is highly grateful to the anonymous referees and Prof. Y. B. Jun, the Editor-in-Chief, for their valuable suggestions. This work was supported by a research grant from the Education Department of Shaanxi Province of China (No. 2010JK831).

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