Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 1, (July 2011), pp. 49- 56
ISSN 2093–9310
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Semi-precompactness in Šostak's L-fuzzy topological spaces

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Received 5 November 2010; Accepted 10 January 2011

Abstract. In this paper, we introduce a good definition of semi-precompactness in L-fuzzy topological spaces in Šostak's sense, where L is a fuzzy lattice. We define this concept on arbitrary L-fuzzy sets, obtain a different characterization and study some of its properties.

2010 AMS Classification: 54A40, 54D30, 53A05

Keywords: L-fuzzy topology, Fuzzy lattice, Semi-preopen set, Semi-precompactness, Good extension.

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1. Introduction

Kubiak [12] and Šostak [20]-[22] introduced the notion of (L-) fuzzy topological spaces as a generalization of L-topological spaces (originally called (L-) fuzzy topological spaces by Chang [6] and Goguen [9]). It is the grade of openness of an L-fuzzy set. A general approach to the study of topological-type structures on fuzzy powersets was developed in [10], [11], [12], [13] and [24].

The notion of compactness is one of the most important concepts in general topology. Therefore, the problem of generalizing classical compactness to fuzzy topological spaces has been intensively discussed over the past 30 years. Many papers on fuzzy compactness have been published and various kinds of fuzzy compactness have been presented and studied. Among these compactness, the fuzzy compactness in L-fuzzy topological spaces introduced by Warner and McLean [23] and extended to arbitrary L-fuzzy sets by Kudri [14] possesses several nice properties, such as: this compactness is defined for arbitrary L-fuzzy sets, is inherited by closed L-fuzzy sets, is preserved under fuzzy continuous functions and arbitrary products, is a good extension and every compact Hausdorff space is regular and normal. Good extensions of some weaker and stronger fuzzy covering properties were introduced and studied by Kudri and Warner.

Aygün et al. [3], introduced the notion of L-fuzzy compactness in L-fuzzy topological spaces in the sense of Šostak as a generalization of the L-fuzzy compactness introduced by Warner and McLean [23]. Based on this definition, various kinds of compactness in L-fuzzy topological spaces in Šostak's sense have been introduced and studied in [1], [2], [3], [4], [5] and [16].

In this paper, a good definition of semi-precompactness on arbitrary fuzzy sets is introduced in L-fuzzy topological spaces in Šostak's sense along the same lines as the L-fuzzy compactness defined by Aygün et al. [3]. We prove the goodness of the definition, obtain a different characterization and study some of its properties.

2. Preliminaries

Throughout this paper X and Y will be non-empty ordinary sets and $L = L(\leq, \vee, \wedge, ')$ will denote a fuzzy lattice, i.e., a completely distributive lattice with a smallest element 0 and largest element 1 ($0 \neq 1$) and with an order reversing involution $a \to a'(a \in L)$ [15]. We shall denote by L^X the lattice of all L-subsets of X and if $A \subseteq X$ by χ_A the characteristic function of A.

Definition 2.1 ([8]). An element p of L is called prime iff $p \neq 1$ and whenever $a, b \in L$ with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$. The set of all prime elements of L will be denoted by $Pr(L)$.

Definition 2.2 ([8]). An element α of L is called union-irreducible or coprime iff whenever $a, b \in L$ with $\alpha \leq a \vee b$ then $\alpha \leq a$ or $\alpha \leq b$. The set of all non-zero union-irreducible elements of L will be denoted by $M(L)$. It is obvious that $p \in Pr(L)$ iff $p' \in M(L)$.

Definition 2.3 ([3]). Let $(X, \mathcal{T})$ be an ordinary topological space. A function $f : (X, \mathcal{T}) \to L$, where L has its Scott topology (topology generated by the sets of the form $\{t \in L : t \not\leq p\}$ where $p \in Pr(L)$ [23]), is said to be Scott continuous iff for every $p \in Pr(L)$, $f^{-1}(\{t \in L : t \not\leq p\}) \in \mathcal{T}$.

Definition 2.4 ([3]). Let $(X, \mathcal{T})$ be an ordinary topological space and $q \in L$. A function $f : (X, \mathcal{T}) \to L$, where L has its Scott topology, is said to be q-Scott continuous iff for every $p \in Pr(L)$ with $q \not\leq p$, $f^{-1}(\{t \in L : t \not\leq p\}) \in \mathcal{T}$.

It is clear that if f is Scott continuous then f is q-Scott continuous for every $q \in L$. Moreover, f is 1-Scott continuous iff f is Scott continuous. Naturally, every function from $(X, \mathcal{T})$ to L is 0-Scott continuous.

Definition 2.5 ([21]). An L-fuzzy topology on X is a map $\mathcal{T} : L^X \to L$ satisfying the following three axioms:

(O1) $\mathcal{T}(\chi_\varnothing)=\mathcal{T}(\chi_X) = 1$,

(O2) $\mathcal{T}(f \wedge g) \geq \mathcal{T}(f) \wedge \mathcal{T}(g)$, for every $f, g \in L^X$,

(O3) $\mathcal{T}(\vee_{i\in I} f_i) \geq \wedge_{i\in I}\mathcal{T}(f_i)$, for every family $(f_i)_{i\in I}$ in L^X.

The pair $(X, \mathcal{T})$ is called an L-fuzzy topological space (L-fts, for short). For every $f \in L^X$, $\mathcal{T}(f)$ is called the degree of openness of the L-fuzzy subset f.

Definition 2.6 ([21]). Let $(X, \mathcal{T})$ be an L-fts. The map $\mathcal{F}_\mathcal{T} : L^X \to L$ defined by $\mathcal{F}_\mathcal{T}(g)= \mathcal{T}(g')$ for every $g \in L^X$ is called the degree of closedness on X.

Definition 2.7 ([7])**.** Let $(X, \mathcal{T})$ be an L-fts and $f \in L^X$.

(1) The closure of f, denoted by $cl(f)$, is defined by

$$cl(f) = \bigwedge\{g \in L^X : \mathcal{T}(g') > 0, f \leq g\}.$$

(2) The interior of f, denoted by $int(f)$, is defined by

$$int(f) = \bigvee\{g \in L^X : \mathcal{T}(g) > 0, g \leq f\}.$$

Definition 2.8 ([17, 18, 19])**.** Let $(X, \mathcal{T})$ be an L-fts and $f \in L^X$.

(1) f is called fuzzy α-open iff for every $p \in Pr(L)$ there exists $g \in L^X$ with $\mathcal{T}(g) \not\leq p$ such that $g \leq f \leq int(cl(g))$.
(2) f is called fuzzy semi-open iff for every $p \in Pr(L)$ there exists $g \in L^X$ with $\mathcal{T}(g) \not\leq p$ such that $g \leq f \leq cl(g)$.
(3) f is called fuzzy preopen iff $f \leq int(cl(f))$.
(4) f is called semi-preopen iff there exists fuzzy preopen set $g \in L^X$ with $g \leq f \leq cl(g)$.

Definition 2.9 ([17, 19, 20])**.** Let $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ be a function. Then,

(1) F is called fuzzy continuous iff $\forall g \in L^X$, $\mathcal{T}(F^{-1}(g)) \geq \mathcal{T}^*(g)$.
(2) F is called fuzzy irresolute iff for each semi-open set $g \in L^X$, $F^{-1}(g)$ is fuzzy semi-open set of X.
(3) F is called fuzzy α-irresolute (resp. pre-irresolute, semipre-irresolute) iff for each α-open (resp. preopen, semi-preopen) set $g \in L^X$, $F^{-1}(g)$ is fuzzy α-open (resp. preopen, semi-preopen) set of X.

Theorem 2.10 ([3])**.** *Let $(X, \mathcal{T})$ be an ordinary topological space. Then, the function $W(\mathcal{T}) : L^X \to L$ defined by $W(\mathcal{T})(f) = \bigvee\{q \in L$: f is q-Scott continuous$\}$, for every $f \in L^X$, is an L-fuzzy topology on X.*

Theorem 2.11 ([3])**.** *If $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ is continuous, then $F : (X, W(\mathcal{T})) \to (Y, W(\mathcal{T}^*))$ is fuzzy continuous.*

Thus, by Theorems 2.10 and 2.11, we obtain an L-fts from a given ordinary topological space, and the functor W from the category TOP of ordinary topological space into the category FTS of L-fts. This provides a "goodness of extension" criterion for L-fuzzy topological properties. An L-fuzzy extension of a topological property of $(X, \mathcal{T})$ is said to be good when it is possessed by the L-fts $(X, W(\mathcal{T}))$ iff the original property is possessed by $(X, \mathcal{T})$.

Lemma 2.12 ([16])**.** *Let $(X, \mathcal{T})$ be a topological space, $f \in L^X$ and $p \in Pr(L)$. Considering the L-fts $(X, W(\mathcal{T}))$ we have:*

(1) $(cl(f))^{-1}(\{t \in L : t \not\leq p\}) \subset cl(f^{-1}(\{t \in L : t \not\leq p\}))$.
(2) $(int(f))^{-1}(\{t \in L : t \not\leq p\}) \subset int(f^{-1}(\{t \in L : t \not\leq p\}))$.

Lemma 2.13 ([16])**.** *Let $(X, \mathcal{T})$ be a topological space and $A \subseteq X$. Considering the L-fts $(X, W(\mathcal{T}))$ we have:*

$$\chi_{cl(A)} = cl(\chi_A) \ \text{ and } \ \chi_{int(A)} = int(\chi_A).$$

Definition 2.14 ([3]). Let $(X, \mathcal{T})$ be an L-fts and $g \in L^X$. g is called compact if for every prime $p \in L$ and every collection $\{f_i\}_{i\in J}$ of L-subsets with $\mathcal{T}(f_i) \not\leq p$ for any $i \in J$ and $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$, there exists a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. If $g = 1_X$, then the L-fts $(X, \mathcal{T})$ is called compact.

In the crisp case of $\mathcal{T}$, fuzzy compactness coincides with the compactness introduced by Warner and McLean [22], and extended to arbitrary L-fuzzy sets by Kudri [13].

Definition 2.15 ([1, 2, 4, 5]). Let $(X, \mathcal{T})$ be an L-fts and $g \in L^X$. g is called α-compact (resp. strongly compact, semi-compact) if for every prime $p \in L$ and every collection $\{f_i\}_{i\in J}$ of α-open (resp. preopen, semi-open) L-subsets such that $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$, there exists a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. If $g = 1_X$, then the L-fts $(X, \mathcal{T})$ is called α-compact (resp. strongly compact, semi-compact).

3. Semi-precompactness and its goodness

In this section, the definition of semi-precompactness on arbitrary fuzzy sets is introduced. We prove the goodness of this semi-precompactness and obtain a different characterization.

Definition 3.1. Let $(X, \mathcal{T})$ be an L-fts and $g \in L^X$. g is called semi-precompact if for every prime $p \in L$ and every collection $\{f_i\}_{i\in J}$ of semi-preopen L-subsets with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$, there exists a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. If $g = 1_X$, then the L-fts $(X, \mathcal{T})$ is called semi-precompact.

Lemma 3.2. *Let $(X, \mathcal{T})$ be an ordinary topological space and $A \subseteq X$. If A is preopen in $(X, \mathcal{T})$ then χ_A is preopen in the L-fts $(X, W(\mathcal{T}))$.*

Proof. Since A is preopen in $(X, \mathcal{T})$ we have $A \subseteq int(cl(A))$. Thus $\chi_A \leq \chi_{int(cl(A))}$ and, by Lemma 2.13, $\chi_{int(cl(A))} = int(cl(\chi_A))$. Therefore $\chi_A \leq int(cl(\chi_A))$. Hence, χ_A is preopen in the L-fts $(X, W(\mathcal{T}))$. □

Lemma 3.3. *Let $(X, \mathcal{T})$ be an ordinary topological space and $A \subseteq X$. If A is semi-preopen in $(X, \mathcal{T})$ then χ_A is semi-preopen in the L-fts $(X, W(\mathcal{T}))$.*

Proof. Since A is semi-preopen in $(X, \mathcal{T})$ there is a preopen set B in $(X, \mathcal{T})$ such that $B \leq A \leq cl(B)$. Thus, $\chi_B \leq \chi_A \leq \chi_{cl(B)} = cl(\chi_B)$ by Lemma 2.13. And by Lemma 3.2, χ_B is preopen in the L-fts $(X, W(\mathcal{T}))$. Hence χ_A is semi-preopen in $(X, W(\mathcal{T}))$. □

Lemma 3.4. *Let $(X, \mathcal{T})$ be an ordinary topological space and $A \subseteq X$. If g is preopen in the L-fts $(X, W(\mathcal{T}))$ and $p \in Pr(L)$, then we have $g_i^{-1}(\{t \in L : t \not\leq p\})$ is preopen in $(X, \mathcal{T})$.*

Proof. Since g is preopen L-subset in the L-fts $(X, W(\mathcal{T}))$ we have $g \leq int(cl(g))$. Then $g^{-1}(\{t \in L : t \not\leq p\}) \subseteq (int(cl(g)))^{-1}(\{t \in L : t \not\leq p\})$. By Lemma 2.12, we have $(int(cl(g)))^{-1}(\{t \in L : t \not\leq p\}) \subseteq int(cl(g^{-1}(\{t \in L : t \not\leq p\})))$. Therefore

$g^{-1}(\{t \in L : t \not\leq p\}) \subseteq int(cl(g^{-1}(\{t \in L : t \not\leq p\})))$. Hence, $g_i^{-1}(\{t \in L : t \not\leq p\})$ is preopen in $(X, \mathcal{T})$. □

We say that a topological space $(X, \mathcal{T})$ is semi-precompact if every semi-preopen cover of X has a finite subcover. In the following theorem we shall prove that semi-precompactness is a good extension.

Theorem 3.5. *Let $(X, \mathcal{T})$ be an ordinary topological space. Then $(X, \mathcal{T})$ is semi-precompact iff the L-fts $(X, W(\mathcal{T}))$ is semi-precompact.*

Proof. Necessity: Let $p \in Pr(L)$ and $\{f_i\}_{i\in J}$ be a family of semi-preopen L-fuzzy sets in $(X, W(\mathcal{T}))$ with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$. Hence for each $x \in X$ there is $i \in J$ such that $f_i(x) \not\leq p$, i.e., $x \in f_i^{-1}(\{t \in L : t \not\leq p\})$. So,

$$X = \cup_{i\in J} f_i^{-1}(\{t \in L : t \not\leq p\}).$$

Because f_i is semi-preopen in $(X, W(\mathcal{T}))$ we have a preopen L-subset g_i in $(X, W(\mathcal{T}))$ such that $g_i \leq f_i \leq cl(g_i)$ for every $i \in J$. Hence, by Lemma 2.12, we get

$$\begin{aligned} g_i^{-1}(\{t \in L : t \not\leq p\}) &\subseteq f_i^{-1}(\{t \in L : t \not\leq p\}) \\ &\subseteq (cl(g_i))^{-1}(\{t \in L : t \not\leq p\}) \\ &\subseteq cl(g_i^{-1}(\{t \in L : t \not\leq p\})). \end{aligned}$$

By Lemma 3.4 we know that $g_i^{-1}(\{t \in L : t \not\leq p\})$ is preopen in $(X, \mathcal{T})$ for each $i \in J$. Then $\{f_i^{-1}(\{t \in L : t \not\leq p\})\}_{i\in J}$ is a semi-preopen cover of $(X, \mathcal{T})$. Since $(X, \mathcal{T})$ is semi-precompact, there is a finite subset F of J such that

$$X = \cup_{i\in F} f_i^{-1}(\{t \in L : t \not\leq p\}), \text{ i.e., } (\vee_{i\in F} f_i)(x) \not\leq p$$

for all $x \in X$. Hence $(X, W(\mathcal{T}))$ is semi-precompact.

Sufficiency: Let $\{A_i\}_{i\in J}$ be a semi-preopen cover of $(X, \mathcal{T})$. Then by Lemma 3.3, $\{\chi_{A_i}\}_{i\in J}$ is a family of semi-preopen L-subsets in $(X, W(\mathcal{T}))$ such that $1 = (\vee_{i\in J} \chi_{A_i})(x) \not\leq p$ for all $x \in X$ and for all $p \in Pr(L)$. Since $(X, W(\mathcal{T}))$ is semi-precompact, there is a finite subset F of J such that $(\vee_{i\in F} \chi_{A_i})(x) \not\leq p$ for all $x \in X$. Hence, $(\vee_{i\in F} \chi_{A_i})(x) = 1$ for all $x \in X$, i.e., $X = \cup_{i\in F} A_i$ and therefore $(X, \mathcal{T})$ is semi-precompact. □

The next theorem provides a different characterization of semi-precompactness.

Theorem 3.6. *Let $(X, \mathcal{T})$ be an L-fts and $g \in L^X$. The L-fuzzy subset g is semi-precompact iff for every $p \in Pr(L)$ and every collection $\{f_i\}_{i\in J}$ of semi-preopen subsets with $(\vee_{i\in J} f_i \vee g')(x) \not\leq p$ for all $x \in X$, there is a finite subset F of J such that $(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$.*

Proof. Necessity: Let $p \in Pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of semi-preopen subsets with $(\vee_{i\in J} f_i \vee g')(x) \not\leq p$ for all $x \in X$. Then $(\vee_{i\in J} f_i \vee g')(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Since g is semi-precompact, there is a finite subset F of J such that $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Take an arbitrary $x \in X$. If $g'(x) \leq p$ then $g'(x) \vee (\vee_{i\in F} f_i)(x) = (\vee_{i\in F} f_i \vee g')(x) \not\leq p$ because $(\vee_{i\in F} f_i)(x) \not\leq p$. If $g'(x) \not\leq p$ then we have $g'(x) \vee (\vee_{i\in F} f_i)(x) = (\vee_{i\in F} f_i \vee g')(x) \not\leq p$. Thus we have $(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$.

Sufficiency: Let $p \in Pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of semi-preopen subsets with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Hence, $(\vee_{i\in J} f_i \vee g')(x) \not\leq p$ for all $x \in X$. From the hypothesis, there is a finite subset F of J such that $(\vee_{i\in F} f_i \vee g')(x) \not\leq p$ for all $x \in X$. Then $(\vee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Thus, g is semi-precompact. □

4. Some properties of the semi-precompactness

In this section, we study some properties of the semi-precompactness.

Theorem 4.1. *Let $(X, \mathcal{T})$ be an L-fts. If h and g are semi-precompact subsets, then $h \vee g$ is semi-precompact as well.*

Proof. Let $p \in Pr(L^X)$ and $(f_i)_{i\in J}$ be a family of semi-preopen sets with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ such that $(h \vee g)(x) \geq p'$. But if $(h \vee g)(x) \geq p'$ then $h(x) \geq p'$ or $g(x) \geq p'$ because $p \in Pr(L^X)$ and we always have if $h(x) \geq p'$ or $g(x) \geq p'$ then $(h \vee g)(x) \geq p'$. From the semi-precompactness of h and g, there are finite subsets F_1, F_2 of J with $(\vee_{i\in F_1} f_i)(x) \not\leq p$ for all $x \in X$ such that $h(x) \geq p'$ and $(\vee_{i\in F_2} f_i)(x) \not\leq p$ for all $x \in X$ such that $g(x) \geq p'$. Then $(\vee_{i\in F_1\cup F_2} f_i)(x) \not\leq p$ for all $x \in X$ such that $h(x) \geq p'$ or $g(x) \geq p'$. Hence $h \vee g$ is semi-precompact. □

Theorem 4.2. *Let $(X, \mathcal{T})$ be an L-fts where X is a finite set. Then $(X, \mathcal{T})$ is semi-precompact.*

Proof. Let $p \in Pr(L^X)$ and $(f_i)_{i\in J}$ be a family of semi-preopen sets with $(\vee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$. Hence, for each $x \in X$ there is $i \in J$ such that

$$x \in f_i^{-1}(\{t \in L : t \not\leq p\}), \text{ i.e., } X = \cup_{i\in J} f_i^{-1}(\{t \in L : t \not\leq p\}).$$

Since X is a finite set, there is a finite subset F of J such that

$$X = \cup_{i\in F} f_i^{-1}(\{t \in L : t \not\leq p\}), \text{ i.e., } (\vee_{i\in F} f_i)(x) \not\leq p$$

for each $x \in X$. So (X, δ) is semi-precompact. □

Theorem 4.3. *Let $(X, \mathcal{T})$ be an L-fts and $g \in L^X$. Then we have the following implications:*

1. *g is semi-precompact* ⇒ (i) *g is strongly compact* ⇒ (ii) *g is α-compact* ⇒ (iii) *g is compact.*
2. *g is semi-precompact* ⇒ (i) *g is semi-compact* ⇒ (ii) *g is α-compact* ⇒ (iii) *g is compact.*

Proof. (1)(i): From Definition 2.8, we know that every pre-open L-fuzzy set is semipreopen, then (i) follows directly from the definitions of semi-precompact and strongly compact. (ii) and (iii) follow from Theorem 4.9 in [5].

(2) (i): From Definition 2.8, we know that every semi-open L-fuzzy set is semipreopen, then (i) follows directly from the definitions of semi-precompact and semi-compact. (ii) and (iii) follow from Theorem 4.10 in [5] and Proposition 5.18 in [1]. □

Theorem 4.4. *Let $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ be a fuzzy irresolute mapping and $g \in L^X$ be semi-precompact in $(X, \mathcal{T})$. Then $F(g)$ is semi-compact in $(Y, \mathcal{T}^*)$.*

Proof. Let $p \in Pr(L^X)$ and $(h_i)_{i\in J}$ be a family of semi-open sets of $(Y, \mathcal{T}^*)$ such that $(\vee_{i\in J} h_i)(y) \not\leq p$ for all $y \in Y$ with $(F(g))(y) \geq p'$. Because f is irresolute, $(F^{-1}(h_i))_{i\in J}$ is a family of semi-open sets of $(X, \mathcal{T})$ such that $(\vee_{i\in J} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. $(F^{-1}(h_i))_{i\in J}$ is also a family of semi-preopen sets of $(X, \mathcal{T})$ such that $(\vee_{i\in J} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Because, if $g(x) \geq p'$, then $F(g)(F(x)) \geq p'$. So, $(\vee_{i\in J} F^{-1}(h_i))(x) = (\vee_{i\in J} h_i)(F(x)) \not\leq p$. From the semi-precompactness of g in $(X, \mathcal{T})$, there exists a finite subset J_0 of J such that $(\vee_{i\in J_0} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. We are going to show that $(\vee_{i\in J_0} h_i)(y) \not\leq p$ for all $y \in Y$ with $(F(g))(y) \geq p'$. In fact, if $(F(g))(y) \geq p'$, then we have $\vee_{x\in F^{-1}(y)} g(x) \geq p'$ which implies that there is $x \in X$ with $g(x) \geq p'$ and $F(x) = y$. Thus, we have $(\vee_{i\in J_0} h_i)(y) = (\vee_{i\in J_0} h_i)(F(x)) = (\vee_{i\in J_0} F^{-1}(h_i))(x) \not\leq p$. This proved that $F(g)$ is semi-compact in $(Y, \mathcal{T}^*)$. □

Corollary 4.5. *Let $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ be a fuzzy irresolute mapping and X be a semi-precompact L-fts. Then $F(X)$ is semi-compact in $(Y, \mathcal{T}^*)$.*

Theorem 4.6. *Let $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ be a fuzzy pre-irresolute mapping and $g \in L^X$ be semi-precompact in $(X, \mathcal{T})$. Then $F(g)$ is strongly compact in $(Y, \mathcal{T}^*)$.*

Proof. Let $p \in Pr(L^X)$ and $(h_i)_{i\in J}$ be a family of preopen sets of $(Y, \mathcal{T}^*)$ such that $(\vee_{i\in J} h_i)(y) \not\leq p$ for all $y \in Y$ with $(F(g))(y) \geq p'$. Because f is pre-irresolute, $(F^{-1}(h_i))_{i\in J}$ is a family of preopen sets of $(X, \mathcal{T})$ such that $(\vee_{i\in J} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. $(F^{-1}(h_i))_{i\in J}$ is also a family of semi-preopen sets of $(X, \mathcal{T})$ such that $(\vee_{i\in J} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Because, if $g(x) \geq p'$, then $F(g)(F(x)) \geq p'$. So,

$$(\vee_{i\in J} F^{-1}(h_i))(x) = (\vee_{i\in J} h_i)(F(x)) \not\leq p.$$

From the semi-precompactness of g in $(X, \mathcal{T})$, there exists a finite subset J_0 of J such that $(\vee_{i\in J_0} F^{-1}(h_i))(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. We are going to show that $(\vee_{i\in J_0} h_i)(y) \not\leq p$ for all $y \in Y$ with $(F(g))(y) \geq p'$. In fact, if $(F(g))(y) \geq p'$, then we have $\vee_{x\in F^{-1}(y)} g(x) \geq p'$ which implies that there is $x \in X$ with $g(x) \geq p'$ and $F(x) = y$. Thus, we have $(\vee_{i\in J_0} h_i)(y) = (\vee_{i\in J_0} h_i)(F(x)) = (\vee_{i\in J_0} F^{-1}(h_i))(x) \not\leq p$. This proved that $F(g)$ is strongly compact in $(Y, \mathcal{T}^*)$. □

Corollary 4.7. *Let $F : (X, \mathcal{T}) \to (Y, \mathcal{T}^*)$ be a fuzzy pre-irresolute mapping and X be a semi-precompact L-fts. Then $F(X)$ is strongly compact in $(Y, \mathcal{T}^*)$.*

Acknowledgements. This work described here is supported by the grants from the National Natural Science Foundation of China (11026108), the Natural Scientific Foundation of Shandong Province (ZR2010AM019) and the Doctor's Foundation of Jinan University (XBS No.0846).

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