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Multi-fuzzy extension of crisp functions using bridge functions

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Abstract. In this paper we investigate various bridge functions like lattice homomorphisms, order homomorphisms, L-fuzzy lattices and strong L-fuzzy lattices for multi-fuzzy extensions of crisp functions and discuss properties of such multi-fuzzy extensions.

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1. Introduction

We introduced multi-fuzzy sets [3, 4] to handle life problems with multi dimensional characteristic properties. The theory of multi-fuzzy sets is an extension of theories of L-fuzzy sets and Atanassov intuitionistic fuzzy sets. We proposed important basic topological, algebraic and set theoretical concepts of multi-fuzzy sets [2, 3, 4, 5, 7] and studied some fundamental properties of them. Multi-fuzzy extension of crisp functions [4] is one of the most relevant part of multi-fuzzy set theory and so we introduced the concept of bridge functions. In this paper, multi-fuzzy extensions of crisp functions based on the bridge functions lattice homomorphisms, order homomorphisms, L-fuzzy lattices and strong L-fuzzy lattices are studied.

2. Preliminaries

Throughout this paper, we will use the following notations. X and Y stand for universal sets, I, J and K stand for indexing sets, L and M stand for partially ordered sets, $\{L_j : j \in J\}$ and $\{M_i : i \in I\}$ are families of complete lattices with order reversing involutions, unless it is stated otherwise and L^X stands for the set of all functions from X to L. The products $\prod M_i$, $\prod L_j$, $\prod N_k$, $\prod M_i^X$, $\prod L_j^Y$ and

$\prod N_k^Z$ vary over $i \in I, j \in J$ and $k \in K$. Partial order $\geq$ is the opposite order relation of the partial order $\leq$.

Definition 2.1 ([1])**.** Let X be a nonempty ordinary set and L be a complete lattice. An L-fuzzy set on X is a mapping $A : X \to L$, that is, the family of all the L-fuzzy sets on X is just L^X consisting of all the mappings from X to L.

Definition 2.2 ([9])**.** Let $' : M \to M$ and $' : L \to L$ be order reversing involutions. A mapping $h : M \to L$ is called an order homomorphism, if it satisfies the conditions $h(0) = 0,\ h(\vee a_i) = \vee h(a_i)$ and $h^{-1}(b') = (h^{-1}(b))'$, where $h^{-1} : L \to M$ is defined by $\forall b \in L,\ h^{-1}(b) = \vee\{a \in M : h(a) \leq b\}$.

Wang [9] proved the following properties of order homomorphism. For every $a \in M$ and $p \in L$; $a \leq h^{-1}(h(a)),\ h(h^{-1}(p)) \leq p,\ h^{-1}(1_L) = 1_M,\ h^{-1}(0_L) = 0_M$ and $a \leq h^{-1}(p)$ if and only if $h(a) \leq p$ if and only if $h^{-1}(p') \leq a'$. Both h and h^{-1} are order preserving and arbitrary join preserving maps. More over $h^{-1}(\wedge a_i) = \wedge h^{-1}(a_i)$.

Definition 2.3 ([10])**.** If $\{L_j : j \in J\}$ is a family of lattices, then the product $\prod L_j$ is a lattice if for arbitrary $x, y \in \prod L_j$, the join $x \vee y$ and the meet $x \wedge y$ of x, y are defined as:

$$(x \vee y)_j = x_j \vee y_j \text{ and } (x \wedge y)_j = x_j \wedge y_j,\ \forall x_j, y_j \in L_j,\ \forall j \in J;$$

or, equivalently, $x \leq y$ is defined by $x_j \leq_j y_j$, $\forall j \in J$, where $\leq$ and $\leq_j$ are the order relations in $\prod L_j$ and L_j respectively.

2.1. **Multi-fuzzy sets**

Definition 2.4 ([3, 4])**.** Let X be a nonempty set, J be an indexing set and $\{L_j : j \in J\}$ a family of partially ordered sets. A multi-fuzzy set A in X is a set :

$$A = \{\langle x, (\mu_j(x))_{j \in J}\rangle : x \in X,\ \mu_j \in L_j^X,\ j \in J\}.$$

The function $\mu_A = (\mu_j)_{j \in J}$ is called the multi-membership function of the multi-fuzzy set A. If $|J| = n$, a natural number, then n is called the dimension of A. Complement of A is $A' = \{\langle x, (\mu'_j(x))_{j \in J}\rangle : x \in X\}$, where μ'_j is the order reversing involution of μ_j.

Definition 2.5 ([3])**.** Let $\{L_j : j \in J\}$ be a family of partially ordered sets,

$$A = \{\langle x, (\mu_j(x))_{j \in J}\rangle : x \in X,\ \mu_j \in L_j^X,\ j \in J\}$$

and

$$B = \{\langle x, (\nu_j(x))_{j \in J}\rangle : x \in X,\ \nu_j \in L_j^X,\ j \in J\}$$

be multi-fuzzy sets in a nonempty set X with the product order. Then $A \sqsubseteq B$ if and only if $\mu_j(x) \leq \nu_j(x)$, $\forall x \in X$ and $\forall j \in J$.

The equality, union and intersection of A and B are defined as:

(a) $A = B$ if and only if $\mu_j(x) = \nu_j(x)$, $\forall x \in X$ and $\forall j \in J$;
(b) $A \sqcup B = \{\langle x, (\mu_j(x) \vee \nu_j(x))_{j \in J}\rangle : x \in X\}$;
(c) $A \sqcap B = \{\langle x, (\mu_j(x) \wedge \nu_j(x))_{j \in J}\rangle : x \in X\}$.

Proposition 2.6 ([3])**.** *Let $A, B, C \in \prod M_i^X$ be multi-fuzzy sets in X. Then*

(a) $A \sqcup A = A,\ A \sqcap A = A$;
(b) $A \sqsubseteq A \sqcup B,\ \ B \sqsubseteq A \sqcup B,\ \ A \sqcap B \sqsubseteq A$ *and* $A \sqcap B \sqsubseteq B$;
(c) $A \sqsubseteq B$ *if and only if* $A \sqcup B = B$ *if and only if* $A \sqcap B = A$.

2.2. **L-fuzzy lattices and strong L-fuzzy lattices**

Definition 2.7 ([8])**.** Let $(M, \wedge_M, \vee_M)$ be a lattice and L be a complete lattice with the least element 0_L and the greatest element 1_L. The mapping $A : M \to L$ is called a lattice-valued fuzzy lattice (L-fuzzy lattice) if all the p-level sets ($p \in L$) of A are sublattices of M.

Proposition 2.8 ([8])**.** *Let $(M, \wedge_M, \vee_M)$ be a lattice and $(L, \wedge_L, \vee_L)$ a complete lattice with 0_L and 1_L.*

(a) *Let $p, q \in L$ and $A : M \to L$ be an L-fuzzy lattice. If $p \le q$, then the q-level set*
$$A_q = \{x \in M : q \le A(x)\}$$
is a sublattice of the p-level set
$$A_p = \{x \in M : p \le A(x)\}.$$
(b) *A mapping $A : M \to L$ is an L-fuzzy lattice if and only if*
$$A(x) \wedge_L A(y) \le A(x \wedge_M y) \ \text{ and } \ A(x) \wedge_L A(y) \le A(x \vee_M y)$$
for all $x, y \in M$.

Definition 2.9 ([6])**.** Let $(M, \wedge, \vee)$ be a lattice and $(L, \wedge, \vee)$ be a lattice with the least element 0_L and the greatest element 1_L. The mapping $A : M \to L$ is called a strong L-fuzzy lattice if $A_a^b = \{x \in M : a \le A(x) \le b\}$ is a sublattice of M, for all $a, b \in L$.

Proposition 2.10 ([6])**.** *Let $A : M \to L$ be a strong L-fuzzy lattice, and let $p, q, r, s \in L$. If $p \le q \le r \le s$, then A_q^r is a sublattice of A_p^s.*

Theorem 2.11 ([6])**.** *Let $(M, \wedge, \vee)$ be a lattice and $(L, \wedge, \vee)$ a complete lattice with 0_L and 1_L. The mapping $A : M \to L$ is a strong L-fuzzy lattice if and only if A satisfies the following conditions, for all $x, y \in M$:*

(a) $A(x) \wedge A(y) \le A(x \wedge y) \le A(x) \vee A(y)$;
(b) $A(x) \wedge A(y) \le A(x \vee y) \le A(x) \vee A(y)$.

Theorem 2.12 ([6])**.** *Let M be a lattice, L be a completely distributive lattice and $A_j : M \to L$ be a strong L-fuzzy lattice (L-fuzzy lattice), for each $j \in J$, then $\bigvee_{j \in J} A_j$ and $\bigwedge_{j \in J} A_j$ are strong L-fuzzy lattices (L-fuzzy lattices respectively).*

Theorem 2.13 ([6])**.** *Let M be a lattice, L be a complete lattice and $A : M \to L$ be a mapping. A is a lattice homomorphism if and only if A is an order preserving strong L-fuzzy lattice.*

Theorem 2.14 ([6])**.** *Let L, M and N be complete lattices, $B : N \to M$ be a lattice homomorphism and $A : M \to L$ be a strong L-fuzzy lattice, then $A \circ B : N \to L$ is a strong L-fuzzy lattice.*

Definition 2.15 ([6]). Let M and L be complete lattices, $A : M \to L$ be an L-fuzzy lattice. Then $A^{-1} : L \to M$ is the upper adjoint of A, that is, for every $b \in L$,

$$A^{-1}(b) = \begin{cases} \bigvee\{a \in M : A(a) \leq b\}, & \text{if there exists an } a \in M \text{ such that } A(a) \leq b; \\ 0_M, & \text{otherwise.} \end{cases}$$

Lemma 2.16 ([6]). *Let M and L be complete lattices, $A : M \to L$ be an L-fuzzy lattice, then:*

(a) $A^{-1}(l_1 \wedge l_2) \leq A^{-1}(l_1) \wedge A^{-1}(l_2)$;
(b) $A^{-1}(l_1 \vee l_2) \geq A^{-1}(l_1) \vee A^{-1}(l_2)$.

That is, A^{-1} is order preserving.

3. Bridge functions and multi-fuzzy extensions

Multi-fuzzy extension of a crisp function is useful to map a multi-fuzzy set into another multi-fuzzy set. In the case of a crisp function, there exists infinitely many multi-fuzzy extensions, even though the domain and range of multi-fuzzy extensions are fixed.

Definition 3.1 ([4]). Let $f : X \to Y$ and $h : \prod M_i \to \prod L_j$ be functions. The multi-fuzzy extension of f and the inverse of the extension are $f : \prod M_i^X \to \prod L_j^Y$ and $f^{-1} : \prod L_j^Y \to \prod M_i^X$ defined by

$$f(A)(y) = \bigvee_{y=f(x)} h(A(x)),\ A \in \prod M_i^X,\ y \in Y$$

and

$$f^{-1}(B)(x) = h^{-1}(B(f(x))),\ B \in \prod L_j^Y,\ x \in X;$$

where h^{-1} is the upper adjoint [9] of h. The function $h : \prod M_i \to \prod L_j$ is called the bridge function of the multi-fuzzy extension of f.

Order homomorphisms, lattice homomorphisms, arbitrary join preserving maps, complement preserving maps, etc. are useful bridge functions for multi-fuzzy extensions.

Example 3.2. Let $h : \prod M_i \to \{0, 1\}$ be the bridge function for the multi-fuzzy extension $f : \prod M_i^X \to 2^X$ of a crisp function $f : X \to X$. This type of extensions is useful in defuzzification problems of multi-fuzzy sets. The crisp subsets

$$\bar{A}_f^h = \left\{ y \in X : \bigvee_{y=f(x)} h(A(x)) = 1 \right\}$$

and

$$\underline{A}_f^h = \left\{ y \in X : \bigwedge_{y=f(x)} h(A(x)) = 1 \right\}$$

satisfy the relation $\underline{A}_f^h \subseteq \bar{A}_f^h$.

Lemma 3.3. *Let $h : \prod M_i \to \{0,1\}$ be the bridge function for the multi-fuzzy extension $f : \prod M_i^X \to 2^X$ of a crisp function $f : X \to X$, and let*

$$\bar{A}_f^h = \left\{ y \in X : \bigvee_{y=f(x)} h(A(x)) = 1 \right\}$$

and

$$\underline{A}_f^h = \left\{ y \in X : \bigwedge_{y=f(x)} h(A(x)) = 1 \right\},$$

for any $A \in \prod M_i^X$. If h is a lattice homomorphism and $A, B \in \prod M_i^X$, then

(a) $\bar{A}_f^h \cup \bar{B}_f^h = (\overline{A \cup B})_f^h$;
(b) $\bar{A}_f^h \cap \bar{B}_f^h = (\overline{A \cap B})_f^h$;
(c) $\underline{A}_f^h \cup \underline{B}_f^h = (\underline{A \cup B})_f^h$;
(d) $\underline{A}_f^h \cap \underline{B}_f^h = (\underline{A \cap B})_f^h$.

Proof. (a) to (d) follow from the definitions. □

Example 3.4. Let $h : \prod M_i \to [0,1]$ be the bridge function for the multi-fuzzy extension $f : \prod M_i^X \to [0,1]^X$ of a crisp function $f : X \to X$. This type of extensions produces fuzzy sets from multi-fuzzy sets and this process is called fuzzification of multi-fuzzy set.

3.1. Extensions based on order homomorphisms

Proposition 3.5 ([4]). *If an order homomorphism $h : \prod M_i \to \prod L_j$ is the bridge function for the multi-fuzzy extension of a crisp function $f : X \to Y$, then for any $k \in K$, $A_k \in \prod M_i^X$, $B_k \in \prod L_j^Y$:*

(a) $f(0_X) = 0_Y$;
(b) $f(\sqcup A_k) = \sqcup f(A_k)$;
(c) $(f^{-1}(B))' = f^{-1}(B')$,

that is, the extension map f is an order homomorphism.

Theorem 3.6 ([4]). *If an order homomorphism $h : \prod M_i \to \prod L_j$ is the bridge function for the multi-fuzzy extension of a crisp function $f : X \to Y$, then for any $k \in K$, $A_k \in \prod M_i^X$, $B_k \in \prod L_j^Y$:*

(a) $A_1 \sqsubseteq A_2$ *implies* $f(A_1) \sqsubseteq f(A_2)$;
(b) $f(\sqcup A_k) = \sqcup f(A_k)$;
(c) $f(\sqcap A_k) \sqsubseteq \sqcap f(A_k)$;
(d) $f(A_{[\alpha]}) \subseteq f(A)_{[h(\alpha)]}$;
(e) $f^{-1}(1_Y) = 1_X$ *and* $f^{-1}(0_Y) = 0_X$;
(f) $B_1 \sqsubseteq B_2$ *implies* $f^{-1}(B_1) \sqsubseteq f^{-1}(B_2)$;
(g) $f^{-1}(\sqcup B_k) = \sqcup f^{-1}(B_k)$;
(h) $f^{-1}(\sqcap B_k) = \sqcap f^{-1}(B_k)$;

(i) $A \sqsubseteq f^{-1}(f(A))$;

(j) $f(f^{-1}(B)) \sqsubseteq B$.

3.2. **Extensions based on lattice valued fuzzy lattices**

Theorem 3.7. *Let $\{L_j : j \in J\}$ and $\{M_i : i \in I\}$ be families of completely distributive lattices, lattice valued fuzzy lattice $h : \prod M_i \to \prod L_j$ be the bridge function for the multi-fuzzy extension of $f : X \to Y$.*

(a) *The supremum extension $\bar{F} : \prod M_i^X \to \prod L_j^Y$ of f defined by;*

$$\bar{F}(A)(y) = \bigvee_{x \in f^{-1}(y)} h(A(x)),\ A \in \prod M_i^X,\ y \in Y,$$

is a lattice valued fuzzy lattice with respect to the inclusion as the order relation.

(b) *The infimum extension $\bar{f} : \prod M_i^X \to \prod L_j^Y$ of f defined by;*

$$\bar{f}(A)(y) = \bigwedge_{x \in f^{-1}(y)} h(A(x)),\ A \in \prod M_i^X,\ y \in Y,$$

is a lattice valued fuzzy lattice with respect to the inclusion as the order relation.

Proof. (a) We have

$$\begin{aligned}
&\bar{F}(A_1 \sqcup A_2)(y) = \bigvee\{h((A_1 \sqcup A_2)(x)) : x \in X,\ y = f(x)\}\\
&= \bigvee\{h(A_1(x) \vee A_2(x)) : x \in X,\ y = f(x)\}\\
&\geq \bigvee\{h(A_1(x)) \wedge h(A_2(x)) : x \in X,\ y = f(x)\}\\
&= \Big(\bigvee\{h(A_1(x)) : x \in X,\ y = f(x)\}\Big) \wedge \Big(\bigvee\{(h(A_2(x)) : x \in X,\ y = f(x)\}\Big)\\
&= \bar{F}(A_1)(y) \wedge \bar{F}(A_2)(y) = (\bar{F}(A_1) \sqcap \bar{F}(A_2))(y).
\end{aligned}$$

That is, $\bar{F}(A_1) \sqcap \bar{F}(A_2) \sqsubseteq \bar{F}(A_1 \sqcup A_2)$. Similarly $\bar{F}(A_1) \sqcap \bar{F}(A_2) \sqsubseteq \bar{F}(A_1 \sqcap A_2)$, which implies that $\bar{F}(A_1) \sqcap \bar{F}(A_2) \sqsubseteq \bar{F}(A_1 \sqcup A_2) \sqcap \bar{F}(A_1 \sqcap A_2)$.

(b) Similar to (a). □

Theorem 3.8. *Let $\{L_j : j \in J\}$ and $\{M_i : i \in I\}$ be families of completely distributive lattices and the strong lattice valued fuzzy lattice $h : \prod M_i \to \prod L_j$ be the bridge function for the multi-fuzzy extension of $f : X \to Y$. Then the supremum and infimum extensions $\bar{F}, \bar{f} : \prod M_i^X \to \prod L_j^Y$ of f are strong lattice valued fuzzy lattices with respect to the inclusion as the order relation.*

Proof. Similar to the Theorem 3.7. □

Remark 3.9. Similarly one can show that, $f(1_X)$ need not equal 1_Y and $f^{-1}(1_Y)$ need not equal 1_X. If an order homomorphism is the bridge function for the multi-fuzzy extension of the crisp function f, then $f(0_X) = 0_Y$, $A \sqsubseteq B$ implies $f(A) \sqsubseteq f(B)$, $f^{-1}(0_Y) = 0_X$, $f(\sqcup A_k) = \sqcup f(A_k)$ and $f(\sqcap A_k) \sqsubseteq \sqcap f(A_k)$.

Theorem 3.10. *Let $f : \prod M_i^X \to \prod L_j^Y$ be the multi-fuzzy extension of a crisp function $f : X \to Y$ with respect to the lattice valued fuzzy lattice $h : \prod M_i \to \prod L_j$ as the bridge function. For any $k \in K,\ A_k \in \prod M_i^X,\ B_k \in \prod L_j^Y$:*

(a) *$B_1 \sqsubseteq B_2$ implies $f^{-1}(B_1) \sqsubseteq f^{-1}(B_2)$;*
(b) *$\sqcup f^{-1}(B_k) \sqsubseteq f^{-1}(\sqcup B_k)$;*
(c) *$f^{-1}(\sqcap B_k) \sqsubseteq \sqcap f^{-1}(B_k)$.*

Proof. (a) Assume that $B_1 \sqsubseteq B_2$. For every $y \in Y$, $B_1(y) \leq B_2(y)$, that is $B_1(f(x)) \leq B_2(f(x)), \forall x \in f^{-1}(y)$. Hence

$$f^{-1}(B_1)(y) = \bigvee h^{-1}(B_1(f(x))) \leq \bigvee h^{-1}(B_2(f(x))) = f^{-1}(B_2)(y).$$

Thus $f^{-1}(B_1) \sqsubseteq f^{-1}(B_2)$.
(b) and (c) follow immediately from (a). □

Remark 3.11. Multi-fuzzy extension of a crisp function with respect to a lattice valued fuzzy lattice (bridge function for the extension) need not satisfy the following relations. For $k \in K$:

(i) $f(0_X) = 0_Y$;
(ii) $f^{-1}(0_Y) = 0_X$;
(iii) $A \sqsubseteq B$ implies $f(A) \sqsubseteq f(B)$;
(iv) $\sqcup f(A_k) \sqsubseteq f(\sqcup A_k)$;
(v) $f(\sqcap A_k) \sqsubseteq \sqcap f(A_k)$.

Proof. See the counter example. Let $L = \{0_L, 1_L\}$, $M = \{0_M, a, b, 1_M\}$ be the diamond lattice with the bottom element 0_M and the top element 1_M. Suppose the L-fuzzy lattice $h : M \to L$ defined by

$$h = \begin{pmatrix} 0_M & a & b & 1_M \\ 1_L & 0_L & 0_L & 1_L \end{pmatrix}$$

is the bridge function for the multi-fuzzy extension $f : M^X \to L^Y$ of the crisp function $f : X \to Y$.

(i) For any $y \in Y$, $f(0_X)(y) = \vee\{h(0_X(x)) : x \in X, y = f(x)\} = h(0_M) = 1_L$. Hence $1_Y = f(0_X) \neq 0_Y$.

(ii) $f^{-1}(0_Y)(y) = h^{-1}(0_Y(f(x))) = h^{-1}(0_L) = a \vee b = 1_M$. Therefore $f^{-1}(0_Y) = 1_X \neq 0_X$.

(iii) Let $A(x) = 0, B(x) = a, \forall x \in X$ and suppose that $A \sqsubseteq B$. Thus $0_M = A(x) \leq B(x) = a, \forall x \in X$. But $1_L = h(0_M) = h(A(x)) \nleq h(B(x)) = h(a) = 0_L$. Hence $f(A) \not\sqsubseteq f(B)$.

(iv) and (v) follow immediately from (iii). □

3.3. **Extensions based on lattice homomorphisms**

Theorem 3.12. *If a complete lattice homomorphism $h : \prod M_i \to \prod L_j$ is the bridge function for the multi-fuzzy extension of a crisp function $f : X \to Y$, then for any $k \in K,\ A_k \in \prod M_i^X,\ B_k \in \prod L_j^Y$:*

(a) *$f(0_X) = 0_Y$ and $f(1_X) = 1_Y$;*
(b) *$A_1 \sqsubseteq A_2$ implies $f(A_1) \sqsubseteq f(A_2)$;*

(c) $f(\sqcup A_k) = \sqcup f(A_k)$;
(d) $f(\sqcap A_k) \sqsubseteq \sqcap f(A_k)$;
(e) $f^{-1}(1_Y) = 1_X$;
(f) $B_1 \sqsubseteq B_2$ *implies* $f^{-1}(B_1) \sqsubseteq f^{-1}(B_2)$;
(g) $\sqcup f^{-1}(B_k) \sqsubseteq f^{-1}(\sqcup B_k)$;
(h) $f^{-1}(\sqcap B_k) \sqsubseteq \sqcap f^{-1}(B_k)$.

Proof. Straightforward. □

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