Annals of Fuzzy Mathematics and Informatics
Volume 1, No. 2, (April 2011), pp. 221- 235
ISSN 2093–9310
http://www.afmi.or.kr



# Credibility approach to solve fuzzy multiple objective minimum cost flow problem

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Received 19 October 2010; Accepted 24 December 2010

Abstract. The aim of this paper is to solve the multiple objective minimum cost flow problem with fuzzy data using credibility approach. Considering data as crisp numbers may not be a true assumption, because data in many real applications cannot be precisely measured. One of the important methods to deal with imprecise data is fuzzy data. We utilize meaning credibility measure, $\alpha$-optimistic and $\alpha$-pessimistic values, to solve the fuzzy version of the multiple objective minimum cost flow problem. Using this approach, the proposed model is transformed to a crisp model. When credibility levels are available and data are trapezoidal or triangular fuzzy numbers, the transformed model is equivalent to a multiple objective minimum cost flow problem with crisp data. In continuation, the concept of $(\alpha, \beta)$-pareto credit is introduced and is shown that the pareto optimal solutions of the crisp model are $(\alpha, \beta)$-pareto credit. Numerical examples illustrate our proposed method.

2010 AMS Classification: 90C70

Keywords: Multiple objective minimum cost flow problem, Fuzzy number, Credibility measure, $(\alpha, \beta)$-pareto credit.

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## 1. Introduction

Minimum cost flow problem is a common problem in combinatorial optimization and network flows. The problem has many applications in practical problems, such as transportation, communication, urban design and job scheduling models and so on [1, 2]. Network flow problems are solved by special methods, which are used even with moderate computing time. However, network flow problems have inherently two ore more conflicting objective functions. For example, the criteria may be minimization of cost for selected routes, minimization of arrival times at the destination points, minimization of deterioration of goods, maximization of safety and etc. So, this is realistic that minimum cost flow problems are considered with multiple objective functions. For a review on minimum cost flow problems and multiple objective minimum cost flow problems, see Hamacher et al. [5].

The classic minimum cost flow problems suppose that all data are exactly known. However, crisp data may not be available, because data in many real applications may be changed in short parts of time. In this case, one of the important methods for discussion with imprecise data is considering fuzzy data in minimum cost flow problems.

Several attempts have been made to consider uncertainty data in minimum cost flow problems. Shih and Lee [11] suggested a fuzzy model of the minimum cost flow problem using multi-level linear programming problem. Liu and Kao [6] considered minimum cost flow problem with fuzzy costs and proposed a ranking function to solve it. Ghatee and Hashemi [4] studied fully fuzzified minimum cost flow problem considering a large variety of ranking functions. Ghatee and Hashemi [3] also investigated some different cases of the fuzzy minimum cost flow problem utilizing a total order and nominal flows.

A common idea to deal with fuzzy data in optimization problems is to convert fuzzy data into interval data utilizing $\alpha$-level sets and Zadeh's extension principle [15], and construct a family of crisp models for the intervals [5]. Therefore, considerable computational efforts in acquiring solutions of problem are needed. Another approach to solve fuzzy optimization problems is the concepts of possibility and necessity measures, which were introduced by Zadeh [14]. In this approach, objective function is changed to a fuzzy constraint using an additional variable. Then, fuzzy constraints are transformed into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers. Then, a crisp program is attained for solving fuzzy program. Although the possibility measure has been widely used, it has no self-duality property [13]. So, $\alpha$-cut method and possibility approach have some difficulties, which were mentioned above. To overcome these difficulties, the concept of credibility approach was defined to solve optimization problems with fuzzy data, which has not the mentioned difficulties. The concept of credibility measure was introduced by Liu and Liu [10]. The concept of credibility theory was extended by Liu [9]. This paper suggests utilizing the credibility measure to solve the minimum cost flow problem and multiple objective minimum cost flow problem.

The rest of the paper is organized as follows. In section 2, briefly is introduced the concept of credibility measure. Section 3 presents the minimum cost flow problem and its fuzzy version. This fuzzy model is transformed to a crisp model using the credibility measure. In the next section, an example is provided to illustrate the proposed method to solve the fuzzy minimum cost flow problem. Multiple objective minimum cost flow problem and its fuzzy version are given in section 5. In this section, the concept of $(\alpha, \beta)$-pareto credit is introduced. It is shown that the pareto optimal solutions of the proposed crisp model by credibility measure are $(\alpha, \beta)$-pareto credit. In the next section, an example of the multiple objective minimum cost flow problem with trapezoidal fuzzy coefficient is provided to explain the proposed method. The last section comprehends our results.

## 2. Credibility measure

Credibility measure was introduced by Liu [8]. Let $X$ be a nonempty set and $P(X)$ the power set of $X$. For any $A \in P(X)$, Liu and Liu [10] defined a credibility measure Cr{A} to express the chance that fuzzy event A occurs. The triplet $(X, P(X), Cr)$ is called a credibility space and fuzzy variable is introduced as a function from this space to the set of real numbers.
Definitions and theorems presented in this section are became from Liu [9].

**Definition 2.1.** $Cr\{.\}$ is a credibility measure if and only if

(i) $Cr\{X\} = 1$
(ii) $if \quad A \subset B \quad Then \quad Cr\{A\} \leq Cr\{B\}$
(iii) $\forall A \in P(X), \quad Cr\{A\} + Cr\{A^c\} = 1$

**Definition 2.2.** Let $\xi$ be a fuzzy variable defined on the credibility space

$$(X, P(X), Cr).$$

Then it's membership function is derived from the credibility measure by

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1, \quad x \in R$$

**Theorem 2.3.** *Let $\xi$ be a fuzzy variable with membership function $\mu$. Then for any set $B$ of real numbers, we have*

$$Cr\{\xi \in B\} = \frac{1}{2}(\sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x))$$

In order to rank fuzzy variables, two critical values optimistic value and pessimistic value are used as follows.

**Definition 2.4.** Let $\xi$ be a fuzzy variable and $\alpha \in (0, 1]$. Then

$$\xi_{\sup}(\alpha) = \sup\{r | Cr\{\xi \geq r\} \geq \alpha\}$$

is called the $\alpha$-optimistic value to $\xi$, and

$$\xi_{\inf}(\alpha) = \inf\{r | Cr\{\xi \leq r\} \geq \alpha\}$$

is called the $\alpha$-pessimistic value to $\xi$.

Consider a fuzzy program as follows

$$\begin{array}{ll} \max & f(x, \xi) \\ s.t & g_j(x, \xi) \leq 0, \quad j = 1, \cdots, p \end{array}$$

where $x$ is a decision vector and $\xi$ is a fuzzy vector. The fuzzy constraints $g_j(x, \xi) \leq 0, \quad (j = 1, \cdots, p)$ do not define a deterministic feasible set. So, utilizing a confidence level $\alpha$ and the concept of credibility measure these fuzzy constraints are transformed to crisp constraints as follows :

$$Cr\{g_j(x, \xi) \leq 0, \quad j = 1, \cdots, p\} \geq \alpha$$

or

$$Cr\{g_j(x, \xi) \leq 0\} \geq \alpha_j, \quad j = 1, \cdots, p$$

where $\alpha$ and $\alpha_j$ $(j = 1, \cdots, p)$ are confidence levels. Liu and Iwamura [7] suggested a spectrum of fuzzy chance-constrained programming (CCP) problem as:

$$\begin{array}{ll} \max\limits_{x} & \max\limits_{\bar{f}} \bar{f} \\ s.t & Cr\{f(x,\xi) \geq \bar{f}\} \geq \beta \\ & Cr\{g_j(x,\xi) \leq 0, \quad j = 1, \cdots, p\} \geq \alpha \end{array} \qquad (1.2)$$

where $\alpha$ and $\beta$ are the predetermined confidence levels and $\max \bar{f}$ is the $\beta$-optimistic value.

If there are multiple objectives functions, then a chance-constrained multiple objective programming problem is as:

$$\begin{array}{ll} \max\limits_{x} & [\max\limits_{\bar{f}_1} \bar{f}_1, \max\limits_{\bar{f}_2} \bar{f}_2, \cdots, \max\limits_{\bar{f}_m} \bar{f}_m] \\ s.t & Cr\{f(x,\xi) \geq \bar{f}_i\} \geq \beta_i, \quad i = 1, \cdots, m \\ & Cr\{g_j(x,\xi) \leq 0, \quad j = 1, \cdots, p\} \geq \alpha \end{array} \qquad (2.2)$$

where $\alpha$ , $\beta_1, \beta_2, \cdots, \beta_m$ are the predetermined confidence levels and $\max \bar{f}_i$ $(i = 1, \cdots, m)$ are the $\beta_i$-optimistic ( $i = 1, \cdots, m$) values to the objective functions.

**Theorem 2.5.** *Assume that the fuzzy vector $\xi$ degenerates to a fuzzy variable $\xi$ with continuous membership function $\mu$ and the function $g(x,\xi)$ has the form $g(x,\xi) = h(x) - \xi$. Then $Cr\{g(x,\xi) \leq 0\} \geq \alpha$ if and only if $h(x) \leq k_\alpha$ where*

$$k_\alpha = \begin{cases} \sup\{K | K = \mu^{-1}(2\alpha)\}, & \text{if } \alpha < \frac{1}{2} \\ \inf\{K | K = \mu^{-1}(2(1-\alpha))\}, & \text{if } \alpha \geq \frac{1}{2}. \end{cases}$$

**Theorem 2.6.** *Assume that the function $g(x,\xi)$ can be rewritten as*

$$g(x,\xi) = h_1(x)\xi_1 + h_2(x)\xi_2 + \cdots + h_t(x)\xi_t + h_0(x)$$

*where $\xi_k$ $(k = 1, \cdots, t)$ are trapezoidal fuzzy variables $(r_k^l, r_k^{ml}, r_k^{mu}, r_k^u)$, $(k = 1, \cdots, t)$ respectively. If two functions $h_k^+(x) = h_k(x) \vee 0$ and $h_k^-(x) = -(h_k(x) \wedge 0)$ $(k = 1, 2, \cdots, t)$ are defined then we have*
(a) $\alpha < \frac{1}{2}, \quad Cr\{g(x,\xi) \leq 0\} \geq \alpha \Leftrightarrow$

$$(1-2\alpha)\Sigma_{k=1}^t [r_k^l h_k^+(x) - r_k^u h_k^-(x)] + 2\alpha\, \Sigma_{k=1}^t [r_k^{ml} h_k^+(x) - r_k^{mu} h_k^-(x)] + h_0(x) \leq 0.$$

(b) $\alpha \geq \frac{1}{2}, \quad Cr\{g(x,\xi) \leq 0\} \geq \alpha \Leftrightarrow$

$$(2-2\alpha)\Sigma_{k=1}^t [r_k^{mu} h_k^+(x) - r_k^{ml} h_k^-(x)] + (2\alpha - 1)\, \Sigma_{k=1}^t [r_k^u h_k^+(x) - r_k^l h_k^-(x)] + h_0(x) \leq 0.$$

## 3. Fuzzy minimum cost flow

Let G(N,A) be a directed network with numbers of edges. Every edge has a cost and a capacity. The cost of each edge must be paid for per unit of flow that goes through the edges. The capacity of each edge is bound on the amount of flow that can go through the edge. Let $c_{ij}$ is considered as costs and a capacity of upper bound is $u_{ij}$ and lower bound is $l_{ij}$. So cost of flow is the summation of $c_{ij}x_{ij}$ for all $(i,j) \in A$. We also let node $i \in N$ possess a number of resources $b(i)$, respectively. The objective is finding a minimum cost flow. The minimum cost flow can be formulated as follows:

$$\begin{aligned}
\min \quad & f(x) = \sum_{(i,j)\in A} c_{ij}x_{ij} \\
s.t \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\
& l_{ij} \le x_{ij} \le u_{ij} \\
& x_{ij} \ge 0, \qquad (i,j) \in A \qquad\qquad (1.3)
\end{aligned}$$

In this paper, minimum cost flow problem is considered with fuzzy parameters. So, in model (1.3) costs and lower and upper bounds are fuzzy numbers. Therefore, model (1.3) can be written as follows:

$$\begin{aligned}
\min \quad & f(x) = \sum_{(i,j)\in A} \tilde{c}_{ij}x_{ij} \\
s.t \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\
& x_{ij} - \tilde{u}_{ij} \le 0 \\
& \tilde{l}_{ij} - x_{ij} \le 0 \\
& x_{ij} \ge 0, \qquad (i,j) \in A \qquad\qquad (2.3)
\end{aligned}$$

where $\tilde{c}_{ij}$ are fuzzy costs and $\tilde{u}_{ij}$ and $\tilde{l}_{ij}$ are fuzzy bounds. Considering the credibility approach, the fuzzy model can be written as:

$$\begin{aligned}
\min_x \quad & \max_{\bar{f}} \bar{f} \\
s.t \quad & Cr\left\{ \sum_{(i,j)\in A} \tilde{c}_{ij}x_{ij} \le \bar{f} \right\} \ge \beta \\
& \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\
& Cr\{x_{ij} - \tilde{u}_{ij} \le 0\} \ge \alpha \\
& Cr\{\tilde{l}_{ij} - x_{ij} \le 0\} \ge \alpha \\
& x_{ij} \ge 0, \qquad (i,j) \in A \qquad\qquad (3.3)
\end{aligned}$$

where $\alpha$, $\beta$ are the predetermined confidence levels. Let $\tilde{c}_{ij} = (c^l_{ij}, c^{ml}_{ij}, c^{mu}_{ij}, c^u_{ij})$, $\tilde{u}_{ij} = (u^l_{ij}, u^{ml}_{ij}, u^{mu}_{ij}, u^u_{ij})$ and $\tilde{l}_{ij} = (l^l_{ij}, l^{ml}_{ij}, l^{mu}_{ij}, l^u_{ij})$ are trapezoidal fuzzy data. At first, $\max \bar{f}$, which is $\beta$-optimistic value to the objective function, is calculated. When $\alpha, \beta \ge \frac{1}{2}$,

$$\max \bar{f} = (2\beta - 1) \sum_{(i,j)\in A} c^l_{ij}x_{ij} + (2 - 2\beta) \sum_{(i,j)\in A} c^{ml}_{ij}x_{ij}$$

Also, using Theorem 6, we have

$$Cr\{x_{ij} - \tilde{u}_{ij} \le 0\} \ge \alpha \Leftrightarrow x_{ij} \le (2\alpha - 1)u^l_{ij} + (2 - 2\alpha)u^{ml}_{ij}$$

$$Cr\{\tilde{l}_{ij} - x_{ij} \leq 0\} \geq \alpha \Leftrightarrow x_{ij} \geq (2\alpha - 1)l_{ij}^{u} + (2 - 2\alpha)l_{ij}^{mu}$$

So, model (3.3) is transformed to the following model:

$$\begin{aligned}
\min \quad & (2\beta - 1)\sum_{\{(i,j)\in A\}} c_{ij}^{l} x_{ij} + (2 - 2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml} x_{ij} \\
s.t \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad \forall i \in N \\
& x_{ij} \leq (2\alpha - 1)u_{ij}^{l} + (2 - 2\alpha)u_{ij}^{ml} \\
& x_{ij} \geq (2\alpha - 1)l_{ij}^{u} + (2 - 2\alpha)l_{ij}^{mu} \\
& x_{ij} \geq 0, \qquad \forall (i,j) \in A \qquad\qquad (4.3)
\end{aligned}$$

When $\alpha, \beta < \frac{1}{2}$, we have

$$\max \bar{f} = (2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu} x_{ij} + (1 - 2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{u} x_{ij}$$

Using Theorem 6, we have

$$Cr\{x_{ij} - \tilde{u}_{ij} \leq 0\} \geq \alpha \Leftrightarrow x_{ij} \leq (1 - 2\alpha)u_{ij}^{u} + 2\alpha u_{ij}^{mu}$$

$$Cr\{\tilde{l}_{ij} - x_{ij} \leq 0\} \geq \alpha \Leftrightarrow x_{ij} \geq (1 - 2\alpha)l_{ij}^{l} + 2\alpha l_{ij}^{ml}$$

So, model (3.3) is converted to the following model as:

$$\begin{aligned}
\min \quad & (2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu} x_{ij} + (1 - 2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{u} x_{ij} \\
s.t \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad \forall i \in N \\
& x_{ij} \leq (1 - 2\alpha)u_{ij}^{u} + 2\alpha u_{ij}^{mu} \\
& x_{ij} \geq (1 - 2\alpha)l_{ij}^{l} + 2\alpha l_{ij}^{ml} \\
& x_{ij} \geq 0, \qquad \forall (i,j) \in A \qquad\qquad (5.3)
\end{aligned}$$

Also, when data are triangular fuzzy numbers, the model (3.3) is transformed to crisp models corresponding to each $\alpha$.

**Example 3.1.** Consider a fuzzy minimum cost flow problem with eight nodes and eleven arces. The data are presented in Table 1. The data were previously studied by Shih and Lee [11].

| Node no | Supply/ demand | Fuzzy cost | Fuzzy lower bound | Fuzzy upper bound | Note |
|---|---|---|---|---|---|
| 1 | 10 | (0.5,1,1.5,2) | (0,0,0,0) | (9,10,11,12) | $x_{21}$ |
| 2 | 20 | (0,0,0.5,1) | (0,0,0.5,1) | (12,13,16,19) | $x_{23}$ |
| 3 | 0 | (5,6,7,8) | (0,0,0,0) | (10,13,14,16) | $x_{26}$ |
| 4 | -5 | (1.5,2,2.5,3) | (0,0,0.25,0.5) | (10,12,14,15) | $x_{14}$ |
| 5 | 0 | (0.5,1,1.5,2) | (0,0,0.5,0.75) | (9,10,10,12) | $x_{34}$ |
| 6 | 0 | (3,4,5,6) | (0,0,0.75,1) | (11,13,15,16) | $x_{35}$ |
| 7 | -15 | (4,5,6,7) | (0,0,0,0) | (11,11,13,15) | $x_{47}$ |
| 8 | -10 | (1.5,2,2.5,3.5) | (0,0,0,0.25) | (6,8,10,12) | $x_{56}$ |
| | | (6,7,8,9) | (0,0,0.5,1) | (5,5,8,10) | $x_{57}$ |
| | | (7,8,9,10) | (0,0,0.25,0.5) | (10,10,12,14) | $x_{68}$ |
| | | (8,9,10,11.5) | (0,0,0,0) | (10,12,14,16) | $x_{21}$ |

Table1 : Parameters for a fuzzy minimum cost flow problem (with 8 nodes and 11 arcs)

The fuzzy minimum cost flow problem can be formulated as:

$$\begin{aligned}
\min\ \ & f_1 = (0.5,1,1.5,2)x_{21} + (0,0,0.5,1)x_{23} + (5,6,7,8)x_{26} + (1.5,2,2.5,3)x_{14} \\
& +(0.5,1,1.5,2)x_{34} + (3,4,5,6)x_{35} + (4,5,6,7)x_{47} + (1.5,2,2.5,3.5)x_{56} \\
& +(6,7,8,9)x_{57} + (7,8,9,10)x_{68} + (8,9,10,11.5)x_{78} \\
& s.t. \\
& x_{14} + x_{21} = 10, \quad x_{21} + x_{23} + x_{26} = 20, \quad x_{34} + x_{35} - x_{23} = 0, \\
& x_{47} - x_{14} - x_{34} = -5, x_{56} + x_{57} - x_{35} = 0, \quad x_{68} - x_{56} - x_{26} = 0, \\
& x_{78} - x_{47} - x_{57} = -15, \quad -x_{68} - x_{78} = -10, \\
& x_{21} \in (0,0,0,0), \quad x_{21} \in (9,10,11,12), \quad x_{23} \in (0,0,0.5,1), \\
& x_{23} \in (12,13,16,19), x_{26} \in (0,0,0,0), \quad x_{26} \in (10,13,14,16), \\
& x_{14} \in (0,0,0.25,0.5), \quad x_{14} \in (10,12,14,15), x_{34} \in (0,0,0.5,0.75), \\
& x_{34} \in (9,10,10,12), \quad x_{35} \in (0,0,0.75,1), \quad x_{35} \in (11,13,15,16), \\
& x_{47} \in (0,0,0,0), \quad x_{47} \in (11,11,13,15), \quad x_{56} \in (0,0,0,0.25), \\
& x_{56} \in (6,8,10,12), x_{57} \in (0,0,0.5,1), \quad x_{57} \in (5,5,8,10), \\
& x_{68} \in (0,0,0.25,0.5), \quad x_{68} \in (1,10,12,14), x_{78} \in (0,0,0,0), \\
& x_{78} \in (10,12,14,16).
\end{aligned} \tag{6.3}$$

We solve this problem with various predetermined confidence levels using model (4.3), when $\alpha, \beta \geq \frac{1}{2}$. The results are demonstrated in Tables 2,3,4 and 5.

| $\beta$ | $f_1$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 0 | 10 | 10 | 10 | 6 | 4 | 11 | 0 | 4 | 10 | 0 |
| 0.7 | 240.20 | 0 | 10 | 10 | 10 | 6 | 4 | 11 | 0 | 4 | 10 | 0 |
| 0.9 | 226.2 | 0 | 13 | 7 | 10 | 6 | 7 | 11 | 3 | 4 | 10 | 0 |
| 1 | 216.5 | 0 | 13 | 7 | 10 | 6 | 7 | 11 | 3 | 4 | 10 | 0 |

Table 2: Solutions with $\alpha = 0.5$

| $\beta$ | $f_1$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 0 | 12.6 | 7.4 | 10 | 6 | 6.6 | 11 | 2.6 | 4 | 10 | 0 |
| 0.7 | 240.24 | 0 | 12.6 | 7.4 | 10 | 6 | 6.6 | 11 | 2.6 | 4 | 10 | 0 |
| 0.9 | 226.36 | 0 | 12.6 | 7.4 | 10 | 6 | 6.6 | 11 | 2.6 | 4 | 10 | 0 |
| 1 | 216.7 | 0 | 12.6 | 7.4 | 10 | 6 | 6.6 | 11 | 2.6 | 4 | 10 | 0 |

Table 3: Solutions with $\alpha = 0.7$

| $\beta$ | $f_1$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 0 | 12.2 | 7.8 | 10 | 5 | 7.2 | 10 | 2.2 | 5 | 10 | 0 |
| 0.7 | 240.28 | 0 | 10.2 | 9.8 | 10 | 6 | 4.2 | 11 | 0.2 | 4 | 10 | 0 |
| 0.9 | 226.52 | 0 | 12.2 | 7.8 | 10 | 6 | 6.2 | 11 | 2.2 | 4 | 10 | 0 |
| 1 | 216.9 | 0 | 12 | 8 | 10 | 5 | 7 | 10 | 2 | 5 | 10 | 0 |

Table 4: Solutions with $\alpha = 0.9$

| $\beta$ | $f_1$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 0 | 12 | 8 | 10 | 5 | 7 | 10 | 2 | 7 | 10 | 0 |
| 0.7 | 240.3 | 0 | 10.25 | 9.75 | 10 | 6 | 4.25 | 11 | 0.25 | 4 | 10 | 0 |
| 0.9 | 226.6 | 0 | 12 | 8 | 10 | 6 | 6 | 11 | 2 | 4 | 10 | 0 |
| 1 | 217 | 0 | 12 | 8 | 10 | 6 | 6 | 11 | 2 | 4 | 10 | 0 |

Table 5: Solutions with $\alpha = 1$

According to Tables 2,3,4 and 5, the amount of flow in arcs 21 and 78 for all various predetermined confidence levels are zero. This can analyze that in the network with such structure, so we do not need to design arcs 21 and 78. The amount of flow is constant in seven arcs. This means that the amount of flow is stable against changing $\alpha$. Also, from Tables 2,3,4 and 5, the more predetermined credit $\alpha, \beta$, the less cost in objective function.

## 4. Fuzzy multiple objective minimum cost flow

The multiple objectives including the economic,the shortness,the environmental and the security indices may be simultaneously considered in minimum cost flow problem. So, multiple objective minimum cost flow problem is formulated as:

$$\begin{aligned} \min \quad & f^k(x) = \sum_{(i,j)\in A} c_{ij}^k x_{ij}, \quad k = 1, \cdots, K \\ s.t \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\ & l_{ij} \leq x_{ij} \leq u_{ij} \\ & x_{ij} \geq 0, \qquad (i,j) \in A \end{aligned} \tag{1.4}$$

In multiple objective minimum cost flow problems often finding the ideal solution that simultaneously maximizes all objectives is impossible. Instead, a solution can be found that is the best tradeoff between the multiple objective. This solution is called pareto solution.

**Definition 4.1.** $x^*$ is a weak pareto optimal solution, if $x^*$ be feasible solution and there not exist any feasible solution as $x$ such that

$$\sum_{(i,j)\in A} c_{ij}^k x_{ij} < \sum_{(i,j)\in A} c_{ij}^k x_{ij}^* \ (k = 1, \cdots, K).$$

**Definition 4.2.** $x^*$ is a strong pareto optimal solution, if $x^*$ be feasible solution and there not exist any feasible solution as $x$ such that

$$\sum_{(i,j)\in A} c_{ij}^k x_{ij} \leq \sum_{(i,j)\in A} c_{ij}^k x_{ij}^* \ (k = 1, \cdots, K)$$

and for at least a $k$, $\sum_{(i,j)\in A} c_{ij}^k x_{ij} < \sum_{(i,j)\in A} c_{ij}^k x_{ij}^*$.

When coefficients of objective functions and upper and lower bounds are fuzzy numbers, we have a fuzzy program as follows:

$$\begin{array}{ll}
\min & f^k(x) = \sum_{(i,j)\in A} \tilde{c}_{ij}^k x_{ij}, \quad k = 1, \cdots, K \\
s.t & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\
& \tilde{l}_{ij} - x_{ij} \leq 0 \\
& x_{ij} - \tilde{u}_{ij} \leq 0 \\
& x_{ij} \geq 0, \qquad (i,j) \in A
\end{array} \tag{2.4}$$

where $\tilde{c}_{ij}^k$ are fuzzy costs and $\tilde{u}_{ij}$ and $\tilde{l}_{ij}$ are fuzzy bounds. Using credibility measure, the fuzzy model is transformed to a credibility model as:

$$\begin{array}{ll}
\min\limits_{x} & \max\limits_{f_k} f_k, \qquad k = 1, \cdots, K \\
s.t & Cr\left\{ \sum_{(i,j)\in A} \tilde{c}_{ij}^k x_{ij} \leq f_k \right\} \geq \beta, \quad k = 1, \cdots, K \\
& \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i \in N \\
& Cr\{\tilde{l}_{ij} - x_{ij} \leq 0\} \geq \alpha \\
& Cr\{x_{ij} - \tilde{u}_{ij} \leq 0\} \geq \alpha \\
& x_{ij} \geq 0, \qquad (i,j) \in A
\end{array} \tag{3.4}$$

When $\alpha, \beta \geq \frac{1}{2}$ and fuzzy data are trapezoidal fuzzy numbers, using appropriate transformations, a crisp model is obtained as:

$$\begin{array}{ll} \min & (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}, \quad k=1,\cdots,K \\ s.t & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i\in N \\ & x_{ij} \leq (2\alpha-1)u_{ij}^{l} + (2-2\alpha)u_{ij}^{ml} \\ & x_{ij} \geq (2\alpha-1)l_{ij}^{u} + (2-2\alpha)l_{ij}^{mu} \\ & x_{ij} \geq 0, \qquad (i,j)\in A \end{array} \qquad (4.4)$$

When $\alpha, \beta < \frac{1}{2}$, the model (3.4) is converted to:

$$\begin{array}{ll} \min & (2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij} + (1-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij}, \quad k=1,\cdots,K \\ s.t & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad i\in N \\ & x_{ij} \leq (1-2\alpha)u_{ij}^{u} + 2\alpha u_{ij}^{mu} \\ & x_{ij} \geq (1-2\alpha)l_{ij}^{l} + 2\alpha l_{ij}^{ml} \\ & x_{ij} \geq 0 \qquad (i,j)\in A \end{array} \qquad (5.4)$$

In continiuation, the concept of pareto optimality is extended to multiple objective minimum cost flow with fuzzy data, when credibility approach is used to solve the model.

**Definition 4.3.** $x$ is $\alpha$-feasible credit, if

(1) $\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i), \quad \forall i \in N$
(2) $Cr\{\tilde{l}_{ij} - x_{ij} \leq 0\} \geq \alpha, \quad \forall (i,j) \in A$
(3) $Cr\{x_{ij} - \tilde{u}_{ij} \leq 0\} \geq \alpha, \quad \forall (i,j) \in A$
(4) $x_{ij} \geq 0, \quad \forall (i,j) \in A.$

**Definition 4.4.** $x^*$ is $(\alpha,\beta)$-weak pareto credit, if $x^*$ be $\alpha$-feasible credit and there not exist any $\alpha$-feasible solution as $x$ such that

$$Cr\left\{\sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij} < \sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij}^{*}\right\} \geq \beta \ (k=1,\cdots,K).$$

**Definition 4.5.** $x^*$ is $(\alpha,\beta)$-strong pareto credit, if $x^*$ be $\alpha$-feasible credit and there not exist any $\alpha$-feasible solution as $x$ such that

$$Cr\left\{\sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij} \leq \sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij}^{*}\right\} \geq \beta \ (k=1,\cdots,K)$$

and for at least a $k$, $Cr\left\{\sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij} < \sum_{(i,j)\in A} \tilde{c}_{ij}^{k}x_{ij}^{*}\right\} \geq \beta.$

**Theorem 4.6.** *The weak pareto optimal solutions in the problem (4.4) are $(\alpha,\beta)$-weak pareto credit.*

*Proof.* Let $x^*$ be weak pareto optimal solution in the problem (4.4). Also, suppose that $x^*$ is not $(\alpha,\beta)$-weak pareto credit. So, there is a $\alpha$-feasible solution as $x$ such that
$Cr\left\{\sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij} < \sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij}^{*}\right\} \geq \beta, k=1,\cdots,K \quad \Longrightarrow$

$$(2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij}$$
$$< (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*}, \quad k=1,\cdots,K$$

From the other side,

$$\Longrightarrow (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}$$
$$< (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij}, \quad k=1,\cdots,K$$
$$\Longrightarrow (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}$$
$$< (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*}, \quad k=1,\cdots,K$$

It contradicts with the fact that $x^*$ is weak pareto solution in the problem (4.4). Therefore, $x^*$ is $(\alpha,\beta)$-weak pareto credit. □

**Theorem 4.7.** *The strong pareto optimal solutions in the model (4.4) are $(\alpha,\beta)$-strong pareto credit.*

*Proof.* Let, $x^*$ is strong pareto optimal solution in the problem (4.5). Also, suppose that $x^*$ is not $(\alpha,\beta)$-strong pareto credit. So, there is a $\alpha$-feasible solution as $x$ such that

$$\begin{cases} Cr\left\{\sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij} \leq \sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij}^{*}\right\} \geq \beta, k=1,\cdots,K \\ \exists k, \quad Cr\{\sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij} < \sum_{(i,j)\in A}\tilde{c}_{ij}^{k}x_{ij}^{*}\} \geq \beta \end{cases}$$

$$\Rightarrow \begin{cases} (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij} \\ \leq (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*}, \quad k=1,\cdots,K \\ \exists k, \quad (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij} \\ < (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*} \end{cases}$$

From the other side,

$$\begin{cases} (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij} \\ \leq (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{u,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{mu,k}x_{ij}, \quad k=1,\cdots,K \end{cases}$$

$$\Rightarrow \begin{cases} (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij} \\ \leq (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*}, \quad k=1,\cdots,K \\ \exists k, \quad (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij} \\ < (2\beta-1)\sum_{\{(i,j)\in A\}} c_{ij}^{l,k}x_{ij}^{*} + (2-2\beta)\sum_{\{(i,j)\in A\}} c_{ij}^{ml,k}x_{ij}^{*} \end{cases}$$

It contradicts with the fact that $x^*$ is strong pareto solution in the problem (4.4). Therefore, $x^*$ is $(\alpha,\beta)$-strong pareto credit. □

Similar Theorems can be stated for the model (5.4).

**Example 4.8.** We consider the example in section 4. Suppose, we have two objectives. The second objective is to minimize the total passing time. The data for this problem is listed in Table 6. This example was previously studied by Shih and Lee [11].

| Node no | Supply/demand | 1nd objective Fuzzy cost | 2nd objective Fuzzy time |
|---|---|---|---|
| 1 | 10 | (0.5,1,1.5,2) | (2,2,2.5,3) |
| 2 | 20 | (0,0,0.5,1) | (1,2,2.5,3) |
| 3 | 0 | (5,6,7,8) | (5,6,7,8) |
| 4 | -5 | (1.5,2,2.5,3) | (2,2,3,3) |
| 5 | 0 | (0.5,1,1.5,2) | (1.2,2,2,2.5) |
| 6 | 0 | (3,4,5,6) | (1.5,2,2,2.5) |
| 7 | -15 | (4,5,6,7) | (6,7,7.5,8) |
| 8 | -10 | (1.5,2,2.5,3.5) | (1,2,2.5,3) |
| | | (6,7,8,9) | (1,2,2.5,3) |
| | | (7,8,9,10) | (2,2.5,3,3.5) |
| | | (8,9,10,11.5) | (2,2.2,3,3.5) |

Table 6: Numerical parameters for example 2

Hence, we have the following fuzzy multiple objective minimum cost flow problem:

$$\begin{aligned}
\min \quad f_1 = {} & (0.5,1,1.5,2)x_{21} + (0,0,0.5,1)x_{23} + (5,6,7,8)x_{26} + (1.5,2,2.5,3)x_{14} \\
& +(0.5,1,1.5,2)x_{34} + (3,4,5,6)x_{35} + (4,5,6,7)x_{47} + (1.5,2,2.5,3.5)x_{56} \\
& +(6,7,8,9)x_{57} + (7,8,9,10)x_{68} + (8,9,10,11.5)x_{78}
\end{aligned}$$

$$\begin{aligned}
\min \quad f_2 = {} & (2,2,2.5,3)x_{21} + (1,2,2.5,3)x_{23} + (5,6,7,8)x_{26} + (2,2,3,3)x_{14} \\
& +(1.2,2,2,2.5)x_{34} + (1.5,2,2,2.5)x_{35} + (6,7,7.5,8)x_{47} + (1,2,2.5,3)x_{56} \\
& +(1,2,2.5,3)x_{57} + (2,2.2,3,3.5)x_{68} + (2,2.2,3,3.5)x_{78}
\end{aligned}$$

$$\begin{aligned}
& s.t. \\
& x_{14} + x_{21} = 10, \quad x_{21} + x_{23} + x_{26} = 20, \quad x_{34} + x_{35} - x_{23} = 0, \\
& x_{47} - x_{14} - x_{34} = -5, x_{56} + x_{57} - x_{35} = 0, \quad x_{68} - x_{56} - x_{26} = 0, \\
& x_{78} - x_{47} - x_{57} = -15, \quad -x_{68} - x_{78} = -10, \\
& x_{21} \in (0,0,0,0), \quad x_{21} \in (9,10,11,12), \quad x_{23} \in (0,0,0.5,1), \\
& x_{23} \in (12,13,16,19), x_{26} \in (0,0,0,0), \quad x_{26} \in (10,13,14,16), \\
& x_{14} \in (0,0,0.25,0.5), \quad x_{14} \in (10,12,14,15), x_{34} \in (0,0,0.5,0.75), \\
& x_{34} \in (9,10,10,12), \quad x_{35} \in (0,0,0.75,1), \quad x_{35} \in (11,13,15,16), \\
& x_{47} \in (0,0,0,0), \quad x_{47} \in (11,11,13,15), \quad x_{56} \in (0,0,0,0.25), \\
& x_{56} \in (6,8,10,12), x_{57} \in (0,0,0.5,1), \quad x_{57} \in (5,5,8,10), \\
& x_{68} \in (0,0,0.25,0.5), \quad x_{68} \in (1,10,12,14), x_{78} \in (0,0,0,0), \\
& x_{78} \in (10,12,14,16). \qquad (6.4)
\end{aligned}$$

The proposed model in previous section is applied to solve the above example. Therefore, the problem is converted to a multiple objective programming problem. Here, ideal solution method is used to solve the obtained multiple objective program. The results of solving this problem with various predetermined confidence levels are reported in Tables 7,8,9 and 10.

| $\beta$ | $f_1$ | $f_2$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 267 | 225 | 0 | 10 | 10 | 10 | 5 | 5 | 10 | 0 | 5 | 10 | 0 |
| 0.7 | 246.2 | 204.6 | 0 | 13 | 7 | 10 | 5 | 8 | 10 | 3 | 5 | 10 | 0 |
| 0.9 | 216.5 | 230.8 | 0 | 13 | 7 | 10 | 5 | 8 | 10 | 3 | 5 | 10 | 0 |
| 1 | 214.1 | 178.7 | 0 | 13 | 7 | 10 | 6 | 7 | 11 | 3 | 4 | 10 | 0 |

Table 7: Solutions with $\alpha = 0.5$

| $\beta$ | $f_1$ | $f_2$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 225 | 0 | 12.6 | 7.4 | 10 | 5 | 7.6 | 10 | 2.6 | 5 | 10 | 0 |
| 0.7 | 246.08 | 204.84 | 0 | 12.6 | 7.4 | 10 | 5 | 7.6 | 10 | 2.6 | 5 | 10 | 0 |
| 0.9 | 219.12 | 189.44 | 0 | 12.6 | 7.4 | 10 | 5 | 7.6 | 10 | 2.6 | 5 | 10 | 0 |
| 1 | 216.7 | 179.3 | 0 | 12.6 | 7.4 | 10 | 5 | 7.6 | 10 | 2.6 | 5 | 10 | 0 |

Table 8: Solutions with $\alpha = 0.7$

| $\beta$ | $f_1$ | $f_2$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 205.08 | 0 | 12.2 | 7.8 | 10 | 6 | 6.2 | 11 | 2.2 | 4 | 10 | 0 |
| 0.7 | 245.88 | 204.84 | 0 | 12.6 | 7.4 | 10 | 5 | 7.6 | 10 | 2.6 | 5 | 10 | 0 |
| 0.9 | 232.12 | 189.92 | 0 | 12.2 | 7.8 | 10 | 5 | 7.2 | 10 | 2.2 | 5 | 10 | 0 |
| 1 | 216.9 | 179.9 | 0 | 12.2 | 7.8 | 10 | 6 | 6.2 | 11 | 2.2 | 4 | 10 | 0 |

Table 9: Solutions with $\alpha = 0.9$

| $\beta$ | $f_1$ | $f_2$ | $x_{21}$ | $x_{23}$ | $x_{26}$ | $x_{14}$ | $x_{34}$ | $x_{35}$ | $x_{47}$ | $x_{56}$ | $x_{57}$ | $x_{68}$ | $x_{78}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 265 | 219.6 | 0 | 12 | 8 | 10 | 6 | 6 | 11 | 2 | 4 | 10 | 0 |
| 0.7 | 245.8 | 205.2 | 0 | 12 | 8 | 10 | 5 | 7 | 10 | 2 | 5 | 10 | 0 |
| 0.9 | 226.6 | 190.16 | 0 | 12 | 8 | 10 | 6 | 6 | 11 | 2 | 4 | 10 | 0 |
| 1 | 222.9 | 181.4 | 0 | 12 | 8 | 10 | 6 | 5 | 11 | 0.2 | 5 | 10 | 0 |

Table 10: Solutions with $\alpha = 1$

Here, for $\alpha = 0.5$ and $\beta = 0.5$, we solve this example. The model is as:

$$\begin{aligned}
\min \quad & f_1 = x_{21} + 6x_{26} + 2x_{14} + x_{34} + 4x_{35} + 5x_{47} + 2x_{56} + 7x_{57} + 8x_{68} + 9x_{78} \\
\min \quad & f_2 = 2x_{21} + 2x_{23} + 6x_{26} + 2x_{14} + 2x_{34} + 2x_{35} + 7x_{47} + 2x_{56} + 2x_{57} \\
& \qquad + 2.2x_{68} + 2.2x_{78}
\end{aligned}$$

$$
\begin{aligned}
&s.t.\\
&x_{14}+x_{21}=10,\quad x_{21}+x_{23}+x_{26}=20,\quad x_{34}+x_{35}-x_{23}=0,\\
&x_{47}-x_{14}-x_{34}=-5, x_{56}+x_{57}-x_{35}=0,\quad x_{68}-x_{56}-x_{26}=0,\\
&x_{78}-x_{47}-x_{57}=-15,\quad -x_{68}-x_{78}=-10,\\
&x_{21}\leq 10,\quad x_{21}\geq 0,\quad x_{23}\geq 0.5,\quad x_{23}\leq 13,\quad x_{26}\geq 0,\quad x_{26}\leq 13,\\
&x_{14}\geq 0.25,\quad x_{14}\leq 12,\quad x_{34}\geq 0.5,\quad x_{34}\leq 10,\quad x_{35}\geq 0.75,\\
&x_{35}\leq 13, x_{47}\geq 0,\quad x_{47}\leq 11, x_{56}\leq 8,\quad x_{56}\geq 0,\quad x_{57}\leq 5,\\
&x_{57}\geq 0.5, x_{68}\leq 10,\quad x_{68}\geq 0.25,\quad x_{78}\geq 0, x_{78}\leq 12 \qquad (7.4)
\end{aligned}
$$

By solving this model, using ideal solution method, the optimal solution is as $x_{21}=0, x_{23}=10, x_{26}=10, x_{14}=10, x_{34}=5, x_{35}=5, x_{47}=10, x_{56}=0, x_{57}=5, x_{58}=10, x_{78}=0$ and $f_1=267, f_2=225$ which is presented in the first row of Table 7. We can test which this solution is $(\alpha,\beta)$-pareto solution, for $\alpha=0.5$ and $\beta=0.5$. Utilizing one of the proposed method [12], in order to pareto optimality test for the obtained solution, we have the following model:

$$
\begin{aligned}
&\max\quad s_1+s_2\\
&s.t.\\
&x_{21}+6x_{26}+2x_{14}+x_{34}+4x_{35}+5x_{47}+2x_{56}+7x_{57}+8x_{68}+9x_{78}+s_1=267\\
&2x_{21}+2x_{23}+6x_{26}+2x_{14}+2x_{34}+2x_{35}+7x_{47}+2x_{56}+2x_{57}+2.2x_{68}\\
&\qquad\qquad +2.2x_{78}+s_2=225\\
&x_{14}+x_{21}=10,\quad x_{21}+x_{23}+x_{26}=20,\quad x_{34}+x_{35}-x_{23}=0,\\
&x_{47}-x_{14}-x_{34}=-5, x_{56}+x_{57}-x_{35}=0,\quad x_{68}-x_{56}-x_{26}=0,\\
&x_{78}-x_{47}-x_{57}=-15,\quad -x_{68}-x_{78}=-10,\\
&x_{21}\leq 10,\quad x_{21}\geq 0,\quad x_{23}\geq 0.5,\quad x_{23}\leq 13,\quad x_{26}\geq 0,\quad x_{26}\leq 13,\\
&x_{14}\geq 0.25,\quad x_{14}\leq 12,\quad x_{34}\geq 0.5,\quad x_{34}\leq 10,\quad x_{35}\geq 0.75,\\
&x_{35}\leq 13,\quad x_{47}\geq 0,\quad x_{47}\leq 11, x_{56}\leq 8,\quad x_{56}\geq 0,\quad x_{57}\leq 5,\\
&x_{57}\geq 0.5,\quad x_{68}\leq 10,\quad x_{68}\geq 0.25,\quad x_{78}\geq 0,\\
&x_{78}\leq 12,\quad s_1\geq 0,\quad s_2\geq 0, \qquad (8.4)
\end{aligned}
$$

Optimal solution for above model is as $s_1=0,\quad s_2=0,\quad x_{21}=0,\quad x_{23}=13,\quad x_{26}=7,\quad x_{14}=10,\quad x_{34}=5.6,\quad x_{35}=7.4,\quad x_{47}=10.6,\quad x_{56}=3,\quad x_{57}=4.4,\quad x_{68}=10,\quad x_{78}=0$. According to the optimal solution in this model, the optimal value of the objective function is zero, therefore the obtained solution of the model (7.4) is pareto solution.

## 5. Conclusions

In this paper, the concept of credibility measure was used for extension multiple objective minimum cost flow problem with fuzzy data. In particular, when data were trapezoidal or triangular fuzzy numbers and confidence levels $\alpha$ and $\beta$ were available, the proposed fuzzy model was transformed to a multiple objective linear programming problem. The concept of $(\alpha,\beta)$-pareto credit was defined and is shown that the optimal solution of crisp program is $(\alpha,\beta)$-pareto credit. At last, illustrative aspect of the proposed method were presented by a numerical example. The more differences between the fuzzy multiple objective minimum cost flow models could be further investigated in the near future. Furthermore, multiple objective minimum cost flow with stochastic data will be considered in the future.

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