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Intuitionistic fuzzy almost semi-generalized closed mappings

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Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy almost semi-generalized closed mappings and intuitionistic fuzzy almost semi-generalized open mappings in intuitionistic fuzzy topological space.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [11] in 1965. Using the concept of fuzzy sets, Chang [2] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [3] defined the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [4] in 1997.

Continuing the work done in the [7], [8], [9], we define the notion of intuitionistic fuzzy almost semi-generalized closed mappings and intuitionistic fuzzy almost semi-generalized open mappings. We discuss characterizations of intuitionistic fuzzy almost semi-generalized closed mappings and open mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic fuzzy closed mappings.

2. Preliminaries

Definition 2.1 ([1]). An *intuitionistic fuzzy set* (IFS, for short) A in X is an object having the form

$$A = \{< x, \mu_A(x), \gamma_A(x) > | x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 ([1]). Let A and B be IFS's of the forms

$A = \{< x, \mu_A(x), \gamma_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \gamma_B(x) > | x \in X\}$.

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\overline{A} = \{< x, \gamma_A(x), \mu_A(x) > | x \in X\}$,
(d) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) > | x \in X\}$,
(e) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) > | x \in X\}$,
(f) $0_\sim = \{< x, 0, 1 > | x \in X\}$ and $1_\sim = \{< x, 1, 0 > | x \in X\}$,
(g) $\overline{\overline{A}} = A$, $\overline{1_\sim} = 0_\sim$, $\overline{0_\sim} = 1_\sim$.

Definition 2.3 ([1]). Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point* (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha, \beta), & \text{if } x = p \\ (0,1), & \text{otherwise.} \end{cases}$$

Definition 2.4 ([3]). An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSí≫s in X satisfying the following axioms:

(i) $0_\sim,\ 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1,\ G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i | i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\overline{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.5 ([3]). Let X and Y are two non empty sets and $f : X \to Y$ be a function. If

$$B = \{< y, \mu_B(y), \gamma_B(y) > | y \in Y\}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) > | x \in X\}.$$

Definition 2.6 ([3]). Let (X, τ) be an IFTS and $A = \{< x, \mu_A(x), \gamma_A(x) > | x \in X\}$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* of A are defined by

$$int(A) = \cup\{G | G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$

$$cl(A) = \cap\{K|K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Note that, for any IFS A in (X, τ), we have $cl(\overline{A}) = \overline{int(A)}$ and $int(\overline{A}) = \overline{cl(A)}$.

Definition 2.7. An IFS $A = \{< x, \mu_A(x), \gamma_A(x) > |x \in X\}$ in an IFTS (X, τ) is called an

(i) *intuitionistic fuzzy semiopen set* (IFSOS) (see [4]) if $A \subseteq cl(int(A))$.

(ii) *intuitionistic fuzzy $\alpha-$open set* (IFαOS) (see [4]) if $A \subseteq int(cl(int(A)))$.

(iii) *intuitionistic fuzzy preopen set* (IFPOS) (see [4]) if $A \subseteq int(cl(A))$.

(iv) *intuitionistic fuzzy regular open set* (IFROS) (see [4]) if $int(cl(A)) = A$.

(v) *intuitionistic fuzzy semi-pre open set* (IFSPOS) (see [6]) if there exists $B \in$ IFPO(X) such that $B \subseteq A \subseteq cl(B)$.

An IFS A is called an *intuitionistic fuzzy semiclosed set*, *intuitionistic fuzzy $\alpha-$closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set* and *intuitionistic fuzzy semi-preclosed set,* respectively (IFSCS, IFαCS, IFPCS, IFRCS and IFSPCS resp), if the complement $\overline{A}$ is an IFSOS, IFαOS, IFPOS, IFROS and IFSPOS respectively. The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy $\alpha-$open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semipreopen) sets of an IFTS (X, τ) is denoted by IFSO(X) (resp IF$_\alpha(X)$, IFPO(X), IFRO(X) and IFSPO(X)).

Definition 2.8 ([7])**.** An IFS A of an IFTS (X, τ) is called an *intuitionistic fuzzy semi-generalized closed* (intuitionistic fuzzy sg-closed) set (IFSGCS) if scl $(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFSOS.

The complement $\overline{A}$ of an intuitionistic fuzzy semi-generalized closed set A is called an intuitionistic fuzzy semi-generalized open (intuitionistic fuzzy sg-open) set (IFSGOS).

Definition 2.9 ([7])**.** An IFTS (X, τ) is said to be an *intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space* if every intuitionistic fuzzy sg-closed set in X is an intuitionistic fuzzy semi-closed in X.

Definition 2.10. A mapping $f : (X, \tau) \to (Y, \kappa)$ from an IFTS (X, τ) into an IFTS (Y, κ) is said to be

(i) an *intuitionistic fuzzy closed mapping* (see [5]) if $f(A)$ is an IFCS in Y, for every IFCS A in X,

(ii) an *intuitionistic fuzzy semi-closed mapping* (see [5]) if $f(A)$ is an IFSCS in Y, for every IFCS A in X,

(iii) an *intuitionistic fuzzy pre-closed mapping* (see [5]) if $f(A)$ is an IFPCS in Y, for every IFCS A in X,

(iv) an *intuitionistic fuzzy $\alpha-$closed mapping* (see [5]) if $f(A)$ is an IFαCS in Y, for every IFCS A in X,

(v) an *intuitionistic fuzzy sg-closed mapping* (see [9]) if $f(A)$ is an IFSGCS in Y, for every IFCS A in X,

(vi) an *intuitionistic fuzzy sg*-closed mapping* (see [9]) if $f(A)$ is an IFSGCS in Y, for every IFSGCS A in X.

Definition 2.11 ([8])**.** A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called an *intuitionistic fuzzy almost sg-continuous mapping* if $f^{-1}(B)$ is an IFSGCS in X, for each IFRCS B in X.

Definition 2.12 ([10])**.** A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called an *intuitionistic fuzzy quasi sg-closed mapping* if $f(B)$ is an IFCS in Y, for each IFSGCS B in X.

3. Intuitionistic fuzzy almost semi-generalized closed mappings

Definition 3.1. A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy almost semi-generalized closed* (intuitionistic fuzzy almost sg-closed) mapping if $f(A)$ is an IFSGCS in Y for every IFRCS A in X.

Example 3.2. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.1}, \frac{b}{0.3}) >, \quad B =< y, (\frac{u}{0.4}, \frac{v}{0.3}), (\frac{u}{0.6}, \frac{v}{0.7}) >$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping.

Theorem 3.3. *Every intuitionistic fuzzy closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption $f(B)$ is an IFCS in Y. In [7], it has been proved that every IFCS is an IFSGCS. Therefore $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example.

Example 3.4. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}) >, \quad B =< y, (\frac{u}{0.3}, \frac{v}{0.1}), (\frac{u}{0.5}, \frac{v}{0.7}) >$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. But $f(\overline{A})$ is not an IFCS in Y, where $\overline{A}$ is an IFCS in X. Therefore f is not an intuitionistic fuzzy closed mapping.

Theorem 3.5. *Every intuitionistic fuzzy semi-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy semi-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption $f(B)$ is an IFSCS in Y. In [7], it has been proved that every IFSCS is an IFSGCS. Therefore $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example.

Example 3.6. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.1}, \frac{b}{0.1}) >, \quad B =< y, (\frac{u}{0.4}, \frac{v}{0.4}), (\frac{u}{0.6}, \frac{v}{0.5}) >$$

Then $\tau = \{0_\sim, 1_\sim, A, B\}$ and $\kappa = \{0_\sim, 1_\sim, C\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. Now

$$f(\overline{A}) =< x, (\frac{u}{0.1}, \frac{v}{0.1}), (\frac{u}{0.5}, \frac{v}{0.4}) >, \quad cl(f(\overline{A})) = \overline{B},$$
$$int(cl(f(\overline{A}))) = int(\overline{B}) = B, \quad int(cl(f(\overline{A}))) = B \nsubseteq f(\overline{A}).$$

Therefore $f(\overline{A})$ is not an IFSCS in Y. Hence f is not an intuitionistic fuzzy semi-closed mapping.

Theorem 3.7. *Every intuitionistic fuzzy $\alpha-$closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy $\alpha-$closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption $f(B)$ is an IFαCS in Y. In [7], it has been proved that every IFαCS is an IFSGCS. Therefore $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example

Example 3.8. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.3}, \frac{b}{0.6}), (\frac{a}{0.1}, \frac{b}{0.3}) >, \quad B =< y, (\frac{u}{0.4}, \frac{v}{0.3}), (\frac{u}{0.6}, \frac{v}{0.7}) >$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. Now

$$f(\overline{A}) =< x, (\frac{u}{0.1}, \frac{v}{0.3}), (\frac{u}{0.3}, \frac{v}{0.6}) >, \quad cl(f(\overline{A})) = 1_\sim,$$
$$int(cl(f(\overline{A}))) = int(1_\sim) = 1_\sim, \quad cl(int(cl(f(\overline{A})))) = 1_\sim \nsubseteq f(\overline{A}).$$

Therefore $f(\overline{A})$ is not an IFαCS in Y. Hence f is not an intuitionistic fuzzy $\alpha-$closed mapping.

Theorem 3.9. *Every intuitionistic fuzzy sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy sg-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example.

Example 3.10. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.2}, \frac{b}{0.2}), (\frac{a}{0.4}, \frac{b}{0.4}) >, \; B =< x, (\frac{a}{0.2}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.4}) >,$$
$$C =< y, (\frac{u}{0.5}, \frac{v}{0.6}), (\frac{u}{0.2}, \frac{v}{0.1}) > .$$

Then $\tau = \{0_\sim, 1_\sim, A, B\}$ and $\kappa = \{0_\sim, 1_\sim, C\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping.

$$IFSOS(Y) = \{0_\sim, 1_\sim, G_{u,v}^{(l_1,m_1),(l_2,m_2)}; l_1 \in [0.5, 1], l_2 \in [0.6, 1], m_1 \in [0, 0.2], m_2 \in [0, 0.1], l_i + m_i \leq 1, i = 1, 2\}$$

where $G_{u,v}^{(l_1,m_1),(l_2,m_2)} =< y, (\frac{u}{l_1}, \frac{v}{l_2}), (\frac{u}{m_1}, \frac{v}{m_2}) >$,

$$IFSCS(Y) = \{0_\sim, 1_\sim, H_{u,v}^{(a_1,b_1),(a_2,b_2)}; a_1 \in [0, 0.2], a_2 \in [0, 0.1], b_1 \in [0.5, 1], b_2 \in [0.6, 1], a_i + b_i \leq 1, i = 1, 2\}$$

where $H_{u,v}^{(l_1,m_1),(l_2,m_2)} =< y, (\frac{u}{l_1}, \frac{v}{l_2}), (\frac{u}{m_1}, \frac{v}{m_2}) >$. Now

$$f(\overline{A}) =< y, (\frac{u}{0.4}, \frac{v}{0.4}), (\frac{u}{0.2}, \frac{v}{0.2}) > \text{ and } scl(f(\overline{A})) = 1_\sim.$$

Then $f(\overline{A}) \subseteq C$, but $scl(f(\overline{A})) \subsetneq C$. Therefore $f(\overline{A})$ is not an IFSGCS in Y. Hence f is not an intuitionistic fuzzy sg-closed mapping.

Theorem 3.11. *Every intuitionistic fuzzy sg*-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy sg*-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFSGCS, B is an IFSGCS in X. By our assumption $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example.

Example 3.12. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.1}, \frac{b}{0.3}) >, \; B =< x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.3}, \frac{b}{0.7}) >,$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. Let $C =< x, (\frac{a}{0.1}, \frac{b}{0.3}), (\frac{a}{0.2}, \frac{b}{0.7}) >$ be an IFSGCS in X.

$$IFSOS(X) = \{0_\sim, 1_\sim, G_{a,b}^{(l_1,m_1),(l_2,m_2)}; l_1 \in [0.2, 1], l_2 \in [0.6, 1], m_1 \in [0, 0.1], m_2 \in [0, 0.3], l_i + m_i \leq 1, i = 1, 2\}$$

where $G_{a,b}^{(l_1,m_1),(l_2,m_2)} =< x, (\frac{a}{l_1}, \frac{b}{l_2}), (\frac{a}{m_1}, \frac{b}{m_2}) >$,

$$IFSCS(X) = \{0_\sim, 1_\sim, H_{a,b}^{(a_1,b_1),(a_2,b_2)}; a_1 \in [0, 0.1], a_2 \in [0, 0.3], b_1 \in [0.2, 1], b_2 \in [0.6, 1], a_i + b_i \leq 1, i = 1, 2\}$$

where $H_{a,b}^{(a_1,b_1),(a_2,b_2)} =< x, (\frac{a}{a_1}, \frac{b}{a_2}), (\frac{a}{b_1}, \frac{b}{b_2}) >$.

$$IFSOS(Y) = \{0_\sim, 1_\sim, K_{u,v}^{(\alpha_1,\beta_1),(\alpha_2,\beta_2)}; \alpha_1 \in [0.4, 1], \alpha_2 \in [0.3, 1], \beta_1 \in [0, 0.3], \beta_2 \in [0, 0.7],\ \alpha_i + \beta_i \leq 1,\ i = 1, 2\}$$

where $K_{u,v}^{(\alpha_1,\beta_1),(\alpha_2,\beta_2)} =< y, (\frac{u}{\alpha_1}, \frac{v}{\alpha_2}), (\frac{u}{\beta_1}, \frac{v}{\beta_2}) >$,

$$IFSCS(Y) = \{0_\sim, 1_\sim, M_{u,v}^{(\gamma_1,\delta_1),(\gamma_2,\delta_2)}; \gamma_1 \in [0, 0.3], \gamma_2 \in [0, 0.7], \delta_1 \in [0.4, 1], \delta_2 \in [0.3, 1],\ \gamma_i + \delta_i \leq 1,\ i = 1, 2\}$$

where $M_{u,v}^{(\gamma_1,\delta_1),(\gamma_2,\delta_2)} =< y, (\frac{u}{\gamma_1}, \frac{v}{\delta_2}), (\frac{u}{\gamma_1}, \frac{v}{\delta_2}) >$. Now $scl(f(C)) = 1_\sim$. Therefore $f(C)$ is not an IFSGCS in Y. Hence f is not an intuitionistic fuzzy sg*-closed mapping.

Theorem 3.13. *Every intuitionistic fuzzy quasi sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let $f : X \to Y$ be an intuitionistic fuzzy quasi sg-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFSGCS, B is an IFSGCS in X. By our assumption $f(B)$ is an IFCS in Y. In [7], it has been proved that every IFCS is an IFSGCS. Therefore $f(B)$ is an IFSGCS in Y. Hence f is an intutitionistic fuzzy almost sg-closed mapping. □

The converse of the above theorem is not true as seen from the following example.

Example 3.14. Let $X = \{a, b\}$, $Y = \{u, v\}$. Let

$$A =< x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.1}, \frac{b}{0.3}) >,\ B =< y, (\frac{u}{0.4}, \frac{v}{0.3}), (\frac{u}{0.6}, \frac{v}{0.7}) >,$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$are IFTSs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\sim, 1_\sim$ are the only IFRCS in X. Now $f(0_\sim) = 0_\sim$ and $f(1_\sim) = 1_\sim$ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. Now $\overline{A}$ is an IFSGCS in X, but $f(\overline{A})$ is not an IFCS in Y. Hence f is not an intuitionistic fuzzy quasi sg-closed mapping.

The relation between various types of intuitionistic fuzzy closed mappings is given in the Figure 1. The reverse implications in the Figure 1 are not true in general.

Definition 3.15. A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy almost semi-generalized open* (intuitionistic fuzzy almost sg-open) mapping if $f(A)$ is an IFSGOS in Y for every IFROS A in X.

Theorem 3.16. *Let $f : X \to Y$ be a mapping. Then the following are equivalent.*
(i) *f is an intuitionistic fuzzy almost sg-closed mapping;*
(ii) *f is an intuitionistic fuzzy almost sg-open mapping.*

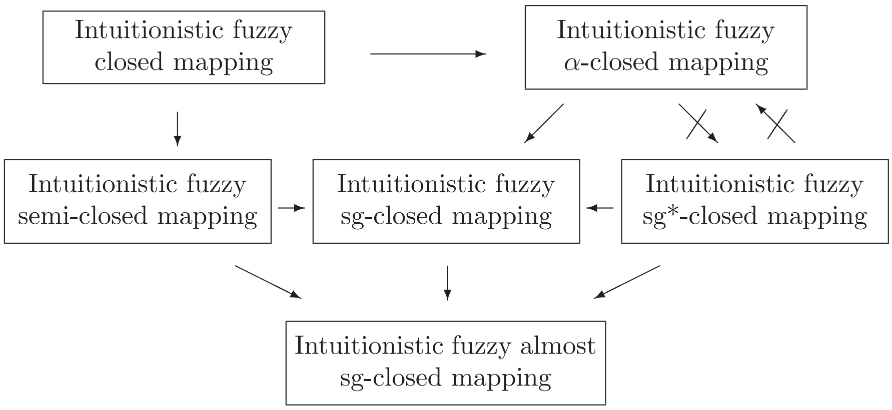

FIGURE 1. The relation between various types of intuitionistic fuzzy closed mappings

Proof. Straightforward. □

Theorem 3.17. *Let $f : X \to Y$ be a mapping where Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space. Then the following are equivalent:*

(i) *f is an intuitionistic fuzzy almost sg-closed mapping;*
(ii) *$scl(f(A)) \subseteq f(cl(A))$ for every IFSPOS A in X;*
(iii) *$scl(f(A)) \subseteq f(cl(A))$ for every IFSOS A in X;*
(iv) *$f(A) \subseteq sint(f(cl(int(A))))$ for every IFPOS A in X.*

Proof. (i)⇒(ii) Let A be an IFSPOS in X. Then $cl(A)$ is an IFRCS in X. By hypothesis $f(cl(A))$ is an IFSGCS in Y. Since Y is an intutitionistic fuzzy semi$-T_{\frac{1}{2}}$ space, $f(cl(A))$ is an IFSCS in Y. Then $scl(f(cl(A))) = f(cl(A))$. Now $scl(f(A)) \subseteq scl(f(cl(A)) = f(cl(A))$. Thus $scl(f(A)) \subseteq f(cl(A))$.

(ii)⇒(iii) Since every IFSOS is an IFSPOS, the proof follows immediately.

(iii)⇒(i) Let A be an IFRCS in X. Then $A = cl(int(A))$, which implies A is an IFSOS in X. By hypothesis, $scl(f(A)) \subseteq f(cl(A)) = f(A) \subseteq scl(f(A))$. Thus $f(A)$ is an IFSCS and hence $f(A)$ is an IFSGCS in Y. Therefore f is an intuitionistic fuzzy almost sg-closed mapping.

(i)⇒(iv) Let A be an IFPOS in X. Then $A \subseteq int(cl(A))$. Since $int(cl(A))$ is an IFROS in X, by our assumption $f(int(cl(A))$ is an IFSGOS in Y. Since Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space, $f(int(cl(A))$ is an IFSOS in Y. Therefore $f(A) \subseteq f(int(cl(A)) = sint(f(int(cl(A))))$.

(iv)⇒(i) Let A be an IFRCS in X. Since every IFRCS is an IFPCS, A is an IFPCS in X. By hypothesis $f(A) \subseteq sint(f(cl(int(A)))) = sint(f(A)) \subseteq f(A)$. This implies $f(A)$ is an IFSOS in Y and hence $f(A)$ is an IFSGOS in Y. Therefore f is an intuitionistic fuzzy almost sg-open mapping. By Theorem 3.16, f is an intuitionistic fuzzy almost sg-closed mapping. □

Definition 3.18. Let $p_{(\alpha,\beta)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an *intuitionistic fuzzy semi-neighborhood* (IFSN) of $p_{(\alpha,\beta)}$, if there exists an IFSOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Theorem 3.19. *Let $f : X \to Y$ be a mapping. Then f is an intuitionistic fuzzy almost sg-closed mapping if for each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFSOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, $scl(f(B))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(\alpha,\beta)} \in Y$.*

Proof. Let $p_{(\alpha,\beta)} \in Y$ and let A be an IFROS in X. Then A is an IFSOS in X. By hypothesis $f^{-1}(p_{(\alpha,\beta)}) \in A$, $p_{(\alpha,\beta)} \in f(A)$ in Y and $scl(f(A))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(\alpha,\beta)}$ in Y. Therefore there exists an IFSOS B in Y such that $p_{(\alpha,\beta)} \in B \subseteq scl(f(A))$. We have $p_{(\alpha,\beta)} \in f(A) \subseteq scl(f(A))$. Now $B = \cup\{p_{(\alpha,\beta)}/p_{(\alpha,\beta)} \in B\} = f(A)$. Therefore $f(A)$ is an IFSOS in Y and hence $f(A)$ is an IFSGOS in Y. Therefore f is an intuitionistic fuzzy almost sg-open mapping and by Theorem 3.16, f is an intuitionistic fuzzy almost sg-closed mapping. □

Theorem 3.20. *Let $f : X \to Y$ be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then $sgcl(f(A)) \subseteq f(cl(A))$ for every IFSPOS A in X.*

Proof. Let A be an IFSPOS in X. Then $cl(A)$ is an IFRCS in X. By hypothesis $f(cl(A))$ is an IFSGCS in Y. Then $sgcl(f(cl(A))) = f(cl(A))$. Now $sgcl(f(A)) \subseteq sgcl(f(cl(A))) = f(cl(A))$. □

Corollary 3.21. *Let $f : X \to Y$ be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then $sgcl(f(A)) \subseteq f(cl(A))$ for every IFSOS A in X.*

Proof. Since every IFSOS is an IFSPOS, the proof follows from the Theorem 3.20. □

Corollary 3.22. *Let $f : X \to Y$ be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then $sgcl(f(A)) \subseteq f(cl(A))$ for every IFPOS A in X.*

Proof. Since every IFSOS is an IFPOS, the proof follows from the Theorem 3.20. □

Theorem 3.23. *Let $f : X \to Y$ be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then $sgcl(f(cl(A))) \subseteq f(cl(spint(A)))$ for every IFSPOS A in X.*

Proof. Let A be an IFSPOS in X. Then $cl(A)$ is an IFRCS in X. By hypothesis, $f(cl(A))$ is an IFSGCS in Y. Then $sgcl(f(cl(A))) = f(cl(A) \subseteq f(cl(spint(A)))$. □

Corollary 3.24. *Let $f : X \to Y$ be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then $sgcl(f(cl(A)) \subseteq f(cl(spint(A)))$ for every IFSOS A in X.*

Proof. Since every IFSOS is an IFSPOS, the proof follows from the above theorem. □

Theorem 3.25. *Let $f : X \to Y$ be a mapping. If $f(sint(B)) \subseteq sint(f(B))$ for every IFS B in X, then f is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let B be an IFROS in X. By hypothesis $f(sint(B)) \subseteq sint(f(B))$. Since every IFROS is an IFSOS, B is an IFSOS in X. Therefore $sint(B) = B$. Hence $f(B) = f(sint(B)) \subseteq sint(f(B)) \subseteq f(B)$. This implies $f(B)$ is an IFSOS in Y. Since every IFSOS is an IFSGOS, f(B) is an IFSGOS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. □

Theorem 3.26. *Let $f : X \to Y$ be a mapping. If $scl(f(B)) \subseteq f(scl(B))$ for every IFS B in X, then f is an intuitionistic fuzzy almost sg-closed mapping.*

Proof. Let B be an IFRCS in X. By hypothesis $scl(f(B)) \subseteq f(scl(B))$. Since every IFRCS is an IFSCS, B is an IFSCS in X. Therefore $scl(B) = B$. Hence $f(B) = f(scl(B)) \supseteq scl(f(B)) \supseteq f(B)$. This implies f(B) is an IFSCS in Y and hence $f(B)$ is an IFSGCS in Y. Thus f is an intuitionistic fuzzy almost sg-closed mapping. □

Theorem 3.27. *Let $f : X \to Y$ be a mapping, where Y is an intutitionistic fuzzy semi$-T_{\frac{1}{2}}$ space. Then the following are equivalent:*

(i) *f is an intuitionistic fuzzy almost sg-open mapping.*

(ii) *for each IFP $p_{(\alpha,\beta)}$ in Y and each IFROS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, $cl(f(cl(B)))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(\alpha,\beta)}$ in Y.*

Proof. (i)⇒(ii) Let $p_{(\alpha,\beta)} \in Y$ and let B be an IFROS in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, $p_{(\alpha,\beta)} \in f(B)$. By hypothesis $f(B)$ is an IFSGOS in Y. Since Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space, $f(B)$ is an IFSOS in Y. Now $p_{(\alpha,\beta)} \in f(B) \subseteq f(cl(B)) \subseteq cl(f(cl(B)))$. Hence $cl(f(cl(B)))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(\alpha,\beta)}$ in Y.

(ii)⇒(i) Let B be an IFOS in X and $f^{-1}(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f(B)$. By hypothesis, $cl(f(cl(B)))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(\alpha,\beta)}$. Therefore there exists an IFSGOS A in Y such that $p_{(\alpha,\beta)} \in A \subseteq cl(fcl(B))$. Now $A = \cup\{p_{(\alpha,\beta)}/p_{(\alpha,\beta)} \in A\} = f(B)$. Therefore $f(B)$ is an IFSOS and hence $f(B)$ is an IFSGOS in Y. Thus f is an intuitionistic fuzzy almost sg-open mapping. □

Theorem 3.28. *Let $f : X \to Y$ be a mapping, where Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space. Then the following statements are equivalent:*

(i) *f is an intuitionistic fuzzy almost sg-closed mapping,*

(ii) *$scl(f(A)) \subseteq f(\alpha cl(A))$ for every IFSPOS A in X,*

(iii) *$scl(f(A)) \subseteq f(\alpha cl(A))$ for every IFSOS A in X,*

(iv) *$f(A) \subseteq sint(f(scl(A)))$ for every IFPOS A in X.*

Proof. (i)⇒(ii) Let A be an IFSPOS in X. Then $cl(A)$ is an IFRCS in X. By hypothesis $f(cl(A))$ is an IFSGCS in Y and hence $f(cl(A))$ is an IFSCS in Y, since Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space. This implies $scl(f(cl(A))) = f(cl(A))$. Now $scl(f(A)) \subseteq scl(f(cl(A)) = f(cl(A))$. Since $cl(A)$ is an IFRCS, we have $cl(int(cl(A))) = cl(A)$. Therefore

$$scl(f(A)) \subseteq f(cl(A)) = f(cl(int(cl(A)))) \subseteq f(A \cup cl(int(cl(A)))) \subseteq f(\alpha cl(A)).$$

Hence $scl(f(A)) \subseteq f(\alpha cl(A))$.

(ii)⇒(iii) Let A be an IFSOS in X. Since every IFSOS is an IFSPOS, the proof is obvious.

(iii)⇒(i) Let A be an IFRCS in X. Then $A = cl(int(A))$. Therefore A is an IFSOS in X. By hypothesis, $scl(f(A)) \subseteq f(\alpha cl(A)) \subseteq f(cl(A)) = f(A) \subseteq scl(f(A))$. Hence $scl(f(A)) = f(A)$. Therefore $f(A)$ is an IFSCS in Y and hence $f(A)$ is an IFSGCS in Y. Thus f is an intuitionistic fuzzy almost sg-closed mapping.

(i)⇒(iv) Let A be an IFPOS in X. Then $A \subseteq int(cl(A))$. Since $int(cl(A))$ is an IFROS in X. By hypothesis $f(int(cl(A)))$ is an IFSGOS in Y. Since Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space, $f(int(cl(A))$ is an IFSOS in Y. Therefore

$$\begin{aligned} f(A) &\subseteq f(cl(int(A)) = sint(f(int(cl(A)))) \\ &= sint(f(A \cup int(cl(A)))) = sint(f(scl(A))). \end{aligned}$$

(iv)⇒(i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis $f(A) \subseteq sint(f(scl(A)))$. This implies that

$$f(A) \subseteq sint(f(A \cup int(cl(A)))) \subseteq sint(f(A \cup A)) = sint(f(A)) \subseteq f(A).$$

Therefore $f(A)$ is an IFSOS in Y and hence $f(A)$ is an IFSGOS in Y. Thus f is an intuitionistic fuzzy almost sg-closed mapping. □

Theorem 3.29. *Let $f : X \to Y$ be a mapping, where Y is an intuitionistic fuzzy semi–$T_{\frac{1}{2}}$ space. If f is an intuitionistic fuzzy almost sg-closed mapping, then $int(cl(f(B))) \subseteq f(scl(B))$ for every IFRCS B in X.*

Proof. Let B be an IFRCS in X. Since f is an intuitionistic fuzzy almost sg-closed mapping, $f(B)$ is an IFSGCS in Y. Since Y is an intuitionistic fuzzy semi$-T_{\frac{1}{2}}$ space, $f(B)$ is an IFSCS in Y. Therefore $scl(f(B)) = f(B)$. Now

$$int(cl(f(B))) \subseteq f(B) \cup int(cl(f(B))) \subseteq scl(f(B)) = f(B) = f(scl(B)).$$

Hence $int(cl(f(B))) \subseteq f(scl(B))$. □

Theorem 3.30. *Let $f : X \to Y$ be a mapping, where Y is an intuitionistic fuzzy semi–$T_{\frac{1}{2}}$ space. If f is an intuitionistic fuzzy almost sg-closed mapping, then $f(sint(B))) \subseteq cl(int(f(B))$ for every IFROS B in X.*

Proof. The proof follows from above theorem by taking complement. □

Theorem 3.31. *Let $f : X \to Y$ be a bijective mapping. Then the following statements are equivalent:*

(i) *f is an intuitionistic fuzzy almost sg-open mapping,*
(ii) *f is an intuitionistic fuzzy almost sg-closed mapping,*
(iii) *f^{-1} is an intuitionistic fuzzy almost sg-continuous mapping.*

Proof. (i)⇒(ii) Obvious.

(ii)⇒(iii) Let A be an IFRCS in X. By our assumption $f(A)$ is an IFSGCS in Y. That is $(f^{-1})^{-1}(A) = f(A)$ is an IFSGCS in Y. Hence f^{-1} is an intuitionistic fuzzy almost sg-continuous mapping.

(iii)⇒(i) Let A be an IFRCS in X. By hypothesis $(f^{-1})^{-1}(A) = f(A)$ is an IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. □

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