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A new approach for solving fully fuzzy multi-choice multi-objective linear programming problem

Shashi Aggarwal, Uday Sharma

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Abstract. In this paper, we have focused on Fully Fuzzy Multi-choice Multi-Objective Linear Programming (FFMMOLP) problem in which all the coefficients and decision variables are trapezoidal fuzzy numbers and all the constraints are fuzzy equality or inequality. A new similarity measure is introduced for trapezoidal fuzzy numbers. Using this similarity measure together with magnitude of trapezoidal fuzzy numbers, a new method for solving FFMMOLP problem is proposed. If Decision Maker (DM) fixes the degree of similarity measure between the two side trapezoidal fuzzy numbers in each constraint, then the fuzzy Pareto-optimal solution of FFMMOLP problem is obtained. In the end, proposed method is illustrated through a numerical example.

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Corresponding Author: Uday Sharma (udaysharma88@yahoo.com)

1. Introduction

In real world, DM does not know precisely the exact value of decision parameters. One of the foremost approaches to treat such situations is fuzzy linear programming based on the concept of fuzzy set theory proposed by Zadeh [35]. In recent years, many researchers have investigated multi-objective linear programming under fuzzy environment [7, 13, 15, 24, 30, 36].

In last few decades, several researchers have shifted their focus to Fully Fuzzy Linear Programming (FFLP) problem as it is easily correlated with the real world problems. Various methods have been proposed in the literature to solve FFLP problem. Lotfi et al. [27] approximated the parameters of FFLP problem to the nearest symmetric triangular fuzzy numbers and found the fuzzy optimal approximate solution. Amit et al. [18] applied ranking function method to convert fuzzy objective of FFLP problem into crisp one and obtained the fuzzy optimal solution of FFLP problem. Kumar and Singh [20] found the fuzzy optimal solution of FFLP problem by converting the FFLP problem into crisp linear programming problem by using ranking function. Khan et al. [22] provided a modified version of simplex method for FFLP problem. Ezzati et al. [14] applied a new lexicographical ordering of triangular fuzzy numbers and converted the FFLP problem into crisp multi-objective linear programming problem in order to find the exact optimal solution of FFLP problem. Kaur and Kumar [23] introduced Mehar's method and applied it to a FFLP problem using L-R fuzzy numbers as parameters. Nasseri et al. [31] proposed a new method for solving FFLP problem by using the concept of memership function. Recently, Cheng et al. [11] solved the FFLP problem through compromise programming and applied the similarity measure on the tolerance level of each constraint. The concept of similarity measure between fuzzy numbers was first introduced by Zwick et al. [37] in 1987. In the literature, several methods based on the concept of geometric distance, grade mean integration, norm, Center of Gravity (COG), height and perimeter of fuzzy numbers for finding the similarity measure of two fuzzy numbers have been investigated by many researchers.[6, 8, 11, 16, 17, 25, 34]

When a problem has multiple objectives, a multi-criteria approach is more suitable by which the decision making can be more precisely captured into an optimization model. Despite the flexibility offered by fuzzy optimization models in the real world problems, to make fuzzy optimization models more realistic recently many researchers have incorporated multi-choice in the fuzzy optimization models as availability of resources, technological coefficients and coefficients of objective functions changes drastically due to some fractious and inevitable circumstances [2, 4, 5, 9, 12, 26, 29, 33]. Biswal and Acharya [4] solved the linear programming problem where the resources could take maximum of eight crisp choices. In 2011, Biswal and Acharya [5] took k number of crisp choices in the resources and combine them using interpolating polynomial.

Cheng et al. [10] solved the Fuzzy Multi-Objective Linear Programming (FMOLP) problem using deviation degree of fuzzy number and obtained the δ-Pareto optimal solution. In this paper, we introduce FFMMOLP problem, in which decision parameters and decision variables are trapezoidal fuzzy numbers and resources, technological coefficients and coefficients of objective functions are multi-choice. Using a proposed similarity measure, FFMMOLP problem is transformed into the Crisp Non-Linear Programming (CNLP) problem and fuzzy Pareto-optimal solution of FFMMOLP problem is obtained. The paper is structured as follows: in Section 2, some basic definitions and arithmetic operations of trapezoidal fuzzy numbers are presented; in Section 3, a review of existing similarity measure is given and a new similarity measure of two trapezoidal fuzzy numbers is defined; formulation of FFM-MOLP problem and a new algorithm for solving this problem is given in Section 4; Section 5 presents a numerical illustration and the paper concludes in Section 6.

2. Preliminaries

In this section, some basic definitions and arithmetic operations of trapezoidal fuzzy numbers related to fuzzy set theory are reviewed.

Definition 2.1 ([3])**.** A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be a Trapezoidal Fuzzy Number (TrFN) if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise.} \end{cases}$$

TrFN(R) denotes the set of all trapezoidal fuzzy numbers.

Definition 2.2 ([3])**.** Let $\tilde{A}$ be a fuzzy set in X and $\alpha \in (0, 1]$. The α-cut of the fuzzy set $\tilde{A}$ is the crisp set given by

$$\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}.$$

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a TrFN then α-cut of $\tilde{A}$ is given by $\tilde{A}_\alpha = [\bar{a}_\alpha, \underline{a}_\alpha]$ for all $0 < \alpha \leq 1$, where $\bar{a}_\alpha = a_1 + (a_2 - a_1)\alpha$ and $\underline{a}_\alpha = a_4 - (a_4 - a_3)\alpha$.

Definition 2.3 ([19])**.** A TrFN $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be a non-negative TrFN if and only if $a_1 \geq 0$.
TrFN$(R)^+$ denotes the set of all non-negative trapezoidal fuzzy numbers.

Definition 2.4 ([19])**.** Two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are said to be equal if and only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$ and $a_4 = b_4$.

Definition 2.5 ([21])**.** The arithmetic operations on two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are given by

(i) $\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
(ii) $\tilde{A} \ominus \tilde{B} = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
(iii) The multiplication of two trapezoidal fuzzy numbers is defined as

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) \cong (c_1, c_2, c_3, c_4)$$

where $c_1 = \min(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4)$, $c_2 = \min(a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3)$, $c_3 = \max(a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3)$ and $c_4 = \max(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4)$.

(iv) The scalar multiplication of TrFN is defined as

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4), & \lambda \geq 0, \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1), & \lambda < 0. \end{cases}$$

Definition 2.6 ([1])**.** Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a TrFN then the magnitude of $\tilde{A}$ is given by

$$\text{Mag}(\tilde{A}) = \frac{1}{2} \int_0^1 (\underline{a}_\alpha + \bar{a}_\alpha + a_2 + a_3) f(\alpha) d\alpha,$$

where $f(\alpha)$ is a non-negative and increasing function on $[0, 1]$ with $f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(\alpha) d\alpha = \frac{1}{2}$.

In this paper we use $f(\alpha) = \alpha$ which gives $\text{Mag}(\tilde{A}) = \frac{a_1 + 5a_2 + 5a_3 + a_4}{12}$.

Definition 2.7 ([1])**.** Let $\tilde{A}$ and $\tilde{B}$ are two trapezoidal fuzzy numbers then the ranking of $\tilde{A}$ and $\tilde{B}$ is defined by magnitude over TrFN as follows:
(i) $\text{Mag}(\tilde{A}) > \text{Mag}(\tilde{B})$ iff $\tilde{A} \succ \tilde{B}$.
(ii) $\text{Mag}(\tilde{A}) < \text{Mag}(\tilde{B})$ iff $\tilde{A} \prec \tilde{B}$.
(iii) $\text{Mag}(\tilde{A}) = \text{Mag}(\tilde{B})$ iff $\tilde{A} \cong \tilde{B}$.

Remark 2.1. $\text{Mag}(\tilde{A}+\tilde{B}) = \text{Mag}(\tilde{A}) + \text{Mag}(\tilde{B})$.

Remark 2.2. $\text{Mag}(k\tilde{A}) = k\,\text{Mag}(\tilde{A})$ for all $k \in R$, where R is the set of real numbers.

3. Similarity measure

In this section, firstly we have reviewed the existing similarity measures between fuzzy numbers. Then, we have defined the new similarity measure between two trapezoidal fuzzy numbers.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.

Chen and Lin [6] proposed the degree of similarity between two trapezoidal fuzzy numbers based on the concept of geometric distance as follows:

$$s(\tilde{A}, \tilde{B}) = 1 - \frac{\sum_{i=1}^{4}(|a_i - b_i|)}{4}$$

Lee [25] defined the similarity measure between two trapezoidal fuzzy numbers using the metric as follows:

$$s(\tilde{A}, \tilde{B}) = 1 - \frac{\|\tilde{A}, \tilde{B}\|_{l_p}}{\|U\|}$$

where $\|\tilde{A} - \tilde{B}\|_{l_p} = \left(\sum_{i=1}^{4}(|a_i - b_i|)^p\right)^{\frac{1}{p}}$, U is universe of discourse and $\|U\| = \max(U) - \min(U)$.

Hsieh and Chen [17] proposed a similarity measure between two trapezoidal numbers with the help of grade mean integration representation as follows:

$$s(\tilde{A}, \tilde{B}) = \frac{1}{1 + d(\tilde{A}, \tilde{B})}$$

where $d(\tilde{A}, \tilde{B}) = |P(\tilde{A}) - P(\tilde{B})|$, $P(\tilde{A}) = \frac{a_1+a_2+a_3+a_4}{6}$ and $P(\tilde{B}) = \frac{b_1+b_2+b_3+b_4}{6}$.

Chen and Chen [8] used the COG of trapezoidal fuzzy number and introduced the similarity measure between two trapezoidal numbers as follows:

$$s(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4}(|a_i - b_i|)}{4}\right]\left[1 - |x_{\tilde{A}} - x_{\tilde{B}}|^{\left\lceil\frac{S_{\tilde{A}}+S_{\tilde{B}}}{2}\right\rceil}\right]\left[\frac{\min(y^*_{\tilde{A}}, y^*_{\tilde{B}})}{\max(y^*_{\tilde{A}}, y^*_{\tilde{B}})}\right]$$

where $y^*_{\tilde{A}} = \begin{cases} \frac{a_3-a_2}{a_4-a_1}+2, & \text{if } a_1 \neq a_4 \\ \frac{1}{2}, & \text{if } a_1 = a_4 \end{cases}$, $\quad x^*_{\tilde{A}} = \frac{y^*_{\tilde{A}}(a_3+a_2)+(a_4+a_1)(1-y^*_{\tilde{A}})}{2}$,

$\left\lceil \frac{S_{\tilde{A}}+S_{\tilde{B}}}{2} \right\rceil = 0$ when $\left\lceil \frac{S_{\tilde{A}}+S_{\tilde{B}}}{2} \right\rceil = 0$ and $\left\lceil \frac{S_{\tilde{A}}+S_{\tilde{B}}}{2} \right\rceil = 1$ when $0 < \left\lceil \frac{S_{\tilde{A}}+S_{\tilde{B}}}{2} \right\rceil \leq 1$, $S_{\tilde{A}} = a_4 - a_1$ and $S_{\tilde{B}} = b_4 - b_1$.

Wei and Chen [34] proposed a similarity measure between two trapezoidal fuzzy numbers based on the perimeters of trapezoidal numbers as follows:

$$s(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4} (|a_i - b_i|)}{4} \right] \left[\frac{\min(P(\tilde{A}), P(\tilde{B})) + 1}{\max(P(\tilde{A}), P(\tilde{B})) + 1} \right]$$

where $P(\tilde{A}) = \sqrt{(a_1 - a_2)^2 + 1} + \sqrt{(a_3 - a_4)^2 + 1} + (a_3 - a_2) + (a_4 - a_1)$ and $P(\tilde{B}) = \sqrt{(b_1 - b_2)^2 + 1} + \sqrt{(b_3 - b_4)^2 + 1} + (b_3 - b_2) + (b_4 - b_1)$.

Hejazi et. al [16] proposed a similarity measure based on the geometric distance, the perimeter of two fuzzy numbers and the area of the two fuzzy numbers as follows:

$$s(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4} (|a_i - b_i|)}{4} \right] \left[\frac{\min(P(\tilde{A}), P(\tilde{B}))}{\max(P(\tilde{A}), P(\tilde{B}))} \right] \left[\frac{\min(A(\tilde{A}), A(\tilde{B})) + 1}{\max(A(\tilde{A}), A(\tilde{B})) + 1} \right]$$

where $P(\tilde{A}) = \sqrt{(a_1 - a_2)^2 + 1} + \sqrt{(a_3 - a_4)^2 + 1} + (a_3 - a_2) + (a_4 - a_1)$,
$P(\tilde{B}) = \sqrt{(b_1 - b_2)^2 + 1} + \sqrt{(b_3 - b_4)^2 + 1} + (b_3 - b_2) + (b_4 - b_1)$,
$A(\tilde{A}) = \frac{(a_3 - a_2 + a_4 - a_1)}{2}$ and $A(\tilde{B}) = \frac{(b_3 - b_2 + b_4 - b_1)}{2}$.

Chen and Lin [6], Chen and Chen [8], Wei and Chen [34] and Hejazi et. al [16] methods can calculate the similarity measure of those trapezoidal fuzzy numbers in which $a_i, b_i \in [0, 1]$ for all $i = 1, 2, 3, 4$. Therefore, all of the above similarity measures cannot be applied on the FFMMOLP problem as the components of TrFNs which are used in the FFMOLP problem are coming from the real line i.e. $a_i, b_i \in \Re$. Hence, we present a new method to determine the similarity measure between two trapezoidal fuzzy numbers using the magnitude of the fuzzy number as the magnitude synthetically reflects the information on every membership degree. The weighting function (i.e.: $f(r)$) in the magnitude of the fuzzy numbers provide liberty to DM for deciding the amount of influence on the different factors which determine similarity measure of two fuzzy numbers. The similarity measure between the two trapezoidal fuzzy numbers is defined as follows:

Definition 3.1. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. The similarity measure between $\tilde{A}$ and $\tilde{B}$ is defined by

$$s(\tilde{A}, \tilde{B}) = \frac{1}{1 + \left| \text{Mag}(\tilde{A}) - \text{Mag}(\tilde{B}) \right|},$$

where $\text{Mag}(\tilde{A}) = \frac{a_1 + 5a_2 + 5a_3 + a_4}{12}$ and $\text{Mag}(\tilde{B}) = \frac{b_1 + 5b_2 + 5b_3 + b_4}{12}$.

Properties of similarity measure. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.

Property 3.1. $0 \leq s(\tilde{A}, \tilde{B}) \leq 1$.

Proof. Clearly, $\left|\text{Mag}(\tilde{A}) - \text{Mag}(\tilde{B})\right| \geq 0$

$$\begin{aligned}
&\Rightarrow \quad 1 + \left|\text{Mag}(\tilde{A}) - \text{Mag}(\tilde{B})\right| \geq 1\\
&\Rightarrow \quad 1 \geq \frac{1}{1 + \left|\text{Mag}(\tilde{A}) - \text{Mag}(\tilde{B})\right|} \geq 0\\
&\Rightarrow \quad 0 \leq s(\tilde{A}, \tilde{B}) \leq 1.
\end{aligned}$$

□

Property 3.2. $s(\tilde{A}, \tilde{B}) = s(\tilde{B}, \tilde{A})$.

Proof. $s(\tilde{A}, \tilde{B}) = \dfrac{1}{1 + \left|\text{Mag}(\tilde{A}) - \text{Mag}(\tilde{B})\right|} = \dfrac{1}{1 + \left|\text{Mag}(\tilde{B}) - \text{Mag}(\tilde{A})\right|} = s(\tilde{B}, \tilde{A})$.

□

Property 3.3. *If* $\tilde{A} = \tilde{B}$ *then* $s(\tilde{A}, \tilde{B}) = 1$.

Proof. Let $\tilde{A} = \tilde{B} \Rightarrow a_i = b_i \ \forall\ i = 1, 2, 3, 4 \Rightarrow \text{Mag}(\tilde{A}) = \text{Mag}(\tilde{B}) \Rightarrow s(\tilde{A}, \tilde{B}) = 1$. □

4. Fully fuzzy multi-choice multi-objective linear programming problem

FFLP problem can be substantially improved by incorporating multiple choices in the resources, technological coefficients and coefficients of the objective functions in order to provide more flexibility in decision making. FFMMOLP problem is the generalization of all the fuzzy linear programming problems. FFMMOLP problem is formulated as follows:

$$\begin{aligned}
\text{(P1)} \quad & \max Z_1(\tilde{X}) = \sum_{j=1}^{n} [(\tilde{c}_{1j1} \text{ or } \tilde{c}_{1j2} \ldots \text{ or } \tilde{c}_{1jt_j^{(1)}}) \otimes \tilde{x}_j]\\
& \quad \vdots\\
& \max Z_k(\tilde{X}) = \sum_{j=1}^{n} [(\tilde{c}_{kj1} \text{ or } \tilde{c}_{kj2} \ldots \text{ or } \tilde{c}_{kjt_j^{(k)}}) \otimes \tilde{x}_j]\\
& \text{subject to}\\
& \quad \sum_{j=1}^{n} [(\tilde{a}_{ij1} \text{ or } \tilde{a}_{ij2} \ldots \text{ or } \tilde{a}_{ijs_j^{(i)}}) \otimes \tilde{x}_j] \preceq \tilde{b}_{i1} \text{ or } \tilde{b}_{i2} \ldots \text{ or } \tilde{b}_{ik_i}, \quad i = 1, 2, \ldots, m_1,\\
& \quad \sum_{j=1}^{n} [(\tilde{a}_{ij1} \text{ or } \tilde{a}_{ij2} \ldots \text{ or } \tilde{a}_{ijs_j^{(i)}}) \otimes \tilde{x}_j] \succeq \tilde{b}_{i1} \text{ or } \tilde{b}_{i2} \ldots \text{ or } \tilde{b}_{ik_i},\\
& \qquad\qquad i = m_1 + 1, m_1 + 2, \ldots, m_2,\\
& \quad \sum_{j=1}^{n} [(\tilde{a}_{ij1} \text{ or } \tilde{a}_{ij2} \ldots \text{ or } \tilde{a}_{ijs_j^{(i)}}) \otimes \tilde{x}_j] \cong \tilde{b}_{i1} \text{ or } \tilde{b}_{i2} \ldots \text{ or } \tilde{b}_{ik_i},\\
& \qquad\qquad i = m_2 + 1, m_2 + 2, \ldots, m,\\
& \quad \tilde{x}_j \in \text{TrFN}(R)^+, \quad j = 1, 2, \ldots, n,
\end{aligned}$$

where $\tilde{X} = [\tilde{x}_j]_{n\times 1}$, $\tilde{c}_{qjt}, \tilde{a}_{ijs}, \tilde{b}_{id} \in \text{TrFN}(R)$ for all $t = 1, 2, \ldots, t_j^{(q)}$, $s = 1, 2, \ldots, s_j^{(i)}$, $d = 1, 2, \ldots, k_i$, $q = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m$.

Let $F(\tilde{X}) = \{\tilde{X} : \tilde{X}$ satisfies all the constraints of (P1)$\}$ be the set of all the fuzzy feasible solutions of (P1).

Mohanaselvi and Ganesan [28] introduced the complete fuzzy optimal solution for fully fuzzy multi-objective linear programming problem. The complete fuzzy optimal solution and fuzzy Pareto-optimal solution for (P1) are defined as follows:

Definition 4.1. A feasible solution $\tilde{X}' \in F(\tilde{X})$ is said to be **complete fuzzy optimal solution** of (P1) if there does not exist any $\tilde{X} \in F(\tilde{X})$ such that

$$\text{Mag}(Z_q(\tilde{X})) \geq \text{Mag}(Z_q(\tilde{X}')) \quad \text{for all } q = 1, 2, \ldots, k$$

A complete optimal solution does not always exist since the objective functions conflict with each other. It is very difficult to maximize all the multiple objective functions simultaneously, thus fuzzy Pareto-optimal solution provide the better compromise solution. Hence, fuzzy Pareto-optimal solution for (P1) is defined as follows:

Definition 4.2. A feasible solution $\tilde{X}' \in F(\tilde{X})$ is said to be **fuzzy Pareto-optimal solution** of (P1) if there does not exist any $\tilde{X} \in F(\tilde{X})$ such that

$$\begin{aligned} &\text{Mag}(Z_q(\tilde{X})) \geq \text{Mag}(Z_q(\tilde{X}')) \quad \text{for all } q = 1, 2, \ldots, k \\ &\qquad\qquad\text{and} \\ &\text{Mag}(Z_w(\tilde{X})) > \text{Mag}(Z_w(\tilde{X}')) \quad \text{for at least one } w = 1, 2, \ldots, k. \end{aligned}$$

Combining all the multi-choices by fuzzy Lagrange interpolating polynomial given by Qian and Hou [32] and converting all the fuzzy inequality constraints into the fuzzy equality constraints by adding fuzzy slack and subtracting fuzzy surplus variables in the fuzzy less than and fuzzy greater than inequality constraints respectively, we get the following problem equivalent to (P1).

Let us denote $(\tilde{X}, \tilde{S}, z_i, u_j^{(q)}, v_j^{(i)})$ by Y.

$$\begin{aligned} \text{(P2)} \quad & \max Z_1(Y) = \sum_{j=1}^{n} \tilde{R}_{1j}(u_j^{(1)}) \\ & \quad \vdots \\ & \max Z_k(Y) = \sum_{j=1}^{n} \tilde{R}_{kj}(u_j^{(k)}) \\ & \text{subject to} \\ & \quad \sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}) \oplus \tilde{s}_i \cong \tilde{P}_i(z_i), \quad i = 1, 2, \ldots, m_1, \\ & \quad \sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}) \ominus \tilde{s}_i \cong \tilde{P}_i(z_i), \quad i = m_1 + 1, m_1 + 2, \ldots, m_2, \\ & \quad \sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}) \cong \tilde{P}_i(z_i), \quad i = m_2 + 1, m_2 + 2, \ldots, m, \\ & \quad z_i = 0, 1, \ldots, (k_i - 1), \quad i = 1, 2, \ldots, m, \\ & \quad u_j^{(q)} = 0, 1, \ldots, (t_j^{(q)} - 1), \quad j = 1, 2, \ldots, n, \ q = 1, 2, \ldots, k, \\ & \quad v_j^{(i)} = 0, 1, \ldots, (s_j^{(i)} - 1), \quad j = 1, 2, \ldots, n, \ i = 1, 2, \ldots, m, \\ & \quad \tilde{x}_j, \tilde{s}_p \in \text{TrFN}(R)^+, \quad p = 1, \ldots, m_2, \ j = 1, 2, \ldots, n, \end{aligned}$$

where

$$\tilde{S} = [\tilde{s}_p]_{m_2\times 1},\ \tilde{s}_p = (s_{p1}, s_{p2}, s_{p3}, s_{p4}),\ p = 1, 2, \ldots, m_2.$$

$$\tilde{P}_i(z_i) = (P_{i1}(z_i), P_{i2}(z_i), P_{i3}(z_i), P_{i4}(z_i)), \quad i = 1, 2, \ldots, m,$$

$$\tilde{Q}_{ij}(v_j^{(i)}) = (Q_{ij1}(v_j^{(i)}), Q_{ij2}(v_j^{(i)}), Q_{ij3}(v_j^{(i)}), Q_{ij4}(v_j^{(i)})), \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m,$$

$$\tilde{R}_{qj}(u_j^{(q)}) = (R_{qj1}(u_j^{(q)}), R_{qj2}(u_j^{(q)}), R_{qj3}(u_j^{(q)}), R_{qj4}(u_j^{(q)})), \quad j = 1, 2, \ldots, n, \quad q = 1, 2, \ldots, k,$$

are the fuzzy Lagrange interpolating polynomials.

$$\begin{aligned} P_{ir}(z_i) = {} & \frac{(z_i-1)(z_i-2)\ldots(z_i-k_i+1)}{(-1)^{(k-1)}(k-1)!}b_{i1r} + \frac{z_i(z_i-2)\ldots(z_i-k_i+1)}{(-1)^{(k-2)}(k-2)!}b_{i2r} \\ & + \frac{z_i(z_i-1)(z_i-3)\ldots(z_i-k_i+1)}{(-1)^{(k-3)}(k-3)!2!}b_{i3r} + \ldots\ldots \\ & + \frac{z_i(z_i-1)(z_i-2)\ldots(z_i-k_i+2)}{(k-1)!}b_{ik_ir}, \quad \forall\ r = 1, 2, 3, 4 \text{ and} \\ & \qquad i = 1, 2, \ldots, m. \end{aligned}$$

$$\begin{aligned} Q_{ijr}(v_j^{(i)}) = {} & \frac{(v_j^{(i)}-1)(v_j^{(i)}-2)\ldots(v_j^{(i)}-s_j^{(i)}+1)}{(-1)^{(s_j^{(i)}-1)}(s_j^{(i)}-1)!}a''_{ij1r} \\ & + \frac{v_j^{(i)}(v_j^{(i)}-2)\ldots(v_j^{(i)}-s_j^{(i)}+1)}{(-1)^{(s_j^{(i)}-2)}(s_j^{(i)}-2)!}a''_{ij2r} \\ & + \frac{v_j^{(i)}(v_j^{(i)}-1)(v_j^{(i)}-3)\ldots(v_j^{(i)}-s_j^{(i)}+1)}{(-1)^{(s_j^{(i)}-3)}(s_j^{(i)}-3)!2!}a''_{ij3r} + \ldots\ldots \\ & + \frac{v_j^{(i)}(v_j^{(i)}-1)(v_j^{(i)}-2)\ldots(v_j^{(i)}-s_j^{(i)}+2)}{(s_j^{(i)}-1)!}a''_{ijs_j^{(i)}r} \end{aligned}$$

where $(a''_{ijb'1}, a''_{ijb'2}, a''_{ijb'3}, a''_{ijb'4}) = \tilde{a}_{ijb'} \otimes \tilde{x}_j$, $\forall\ r = 1, 2, 3, 4$, $b' = 1, 2, \ldots, s_j^{(i)}$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m$,

$$\begin{aligned} R_{qjr}(u_j^{(q)}) = {} & \frac{(u_j^{(q)}-1)(u_j^{(q)}-2)\ldots(u_j^{(q)}-t_j^{(q)}+1)}{(-1)^{(t_j^{(q)}-1)}(t_j^{(q)}-1)!}c'_{qj1r} \\ & + \frac{u_j^{(q)}(u_j^{(q)}-2)\ldots(u_j^{(q)}-t_j^{(q)}+1)}{(-1)^{(t_j^{(q)}-2)}(t_j^{(q)}-2)!}c'_{qj2r} \end{aligned}$$

$$+\frac{u_j^{(q)}(u_j^{(q)}-1)(u_j^{(q)}-3)\dots(u_j^{(q)}-t_j^{(q)}+1)}{(-1)^{(t_j^{(q)}-3)}(t_j^{(q)}-3)!2!}c'_{qt_j^{(q)}3r}+\dots\dots$$
$$+\frac{u_j^{(q)}(u_j^{(q)}-1)(u_j^{(q)}-2)\dots(u_j^{(q)}-t_j^{(q)}+2)}{(t_j^{(q)}-1)!}c'_{qt_j^{(q)}r},$$

where $(c'_{qja'1}, c'_{qja'2}, c'_{qja'3}, c'_{qja'4}) = \tilde{c}_{qja'} \otimes \tilde{x}_j,\ \forall\ r = 1,2,3,4,\ a' = 1,2,\dots,t_j^{(q)}$, $j = 1,2,\dots,n$, and $q = 1,2,\dots,k$.

4.1. **Procedure to find the fuzzy Pareto-optimal solution of (P1).**

In this subsection, we introduce a new algorithm to find the fuzzy Pareto-optimal solution of (P1). The steps of the proposed algorithm are given as follows:

Step 1: Convert (P2) into the Crisp Multi-Objective Non-Linear Programming (CMONLP) problem by applying the similarity measure (Definition 3.1) on the constraints and magnitude (Definition 2.6) on the objective functions. Then (P2) can be written as:

$$\text{(P3)}\quad \max[\text{Mag}(Z_1(Y))] = \text{Mag}\left(\sum_{j=1}^{n}\tilde{R}_{1j}(u_j^{(1)})\right)$$
$$\vdots$$
$$\max[\text{Mag}(\text{Z}_k(Y))] = \text{Mag}\left(\sum_{j=1}^{n}\tilde{R}_{kj}(u_j^{(k)})\right)$$

subject to

$$s\left[\sum_{j=1}^{n}\tilde{Q}_{ij}(v_j^{(i)})\oplus\tilde{s}_i, \tilde{P}_i(z_i)\right] \geq \delta_i,\quad i = 1,2,\dots,m_1,$$
$$s\left[\sum_{j=1}^{n}\tilde{Q}_{ij}(v_j^{(i)})\ominus\tilde{s}_i, \tilde{P}_i(z_i)\right] \geq \delta_i,\quad i = m_1+1, m_1+2,\dots,m_2,$$
$$s\left[\sum_{j=1}^{n}\tilde{Q}_{ij}(v_j^{(i)}), \tilde{P}_i(z_i)\right] \geq \delta_i,\quad i = m_2+1, m_2+2,\dots,m,$$
$$z_i = 0,1\dots,(k_i-1),\quad i = 1,2,\dots,m,$$
$$u_j^{(q)} = 0,1\dots,(t_j^{(q)}-1),\quad j = 1,2,\dots,n,\ q = 1,2\dots,k,$$
$$v_j^{(i)} = 0,1\dots,(s_j^{(i)}-1),\quad j = 1,2,\dots,n,\ i = 1,2,\dots,m,$$
$$x_{j1}, (x_{j2}-x_{j1}), (x_{j3}-x_{j2}), (x_{j4}-x_{j3}) \geq 0,\quad j = 1,2,\dots,n,$$
$$s_{p1}, (s_{p2}-s_{p1}), (s_{p3}-s_{p2}), (s_{p4}-s_{p3}) \geq 0,\quad p = 1,2,\dots,m_2$$

where δ_i is the allowed similarity measure of ith constraint which is provided by DM.

Step 2: Find the membership function of the qth $(q = 1,2,\dots,k)$ crisp objective function of (P3) as follows:

Step 2(a): Find the ideal and anti-ideal values, which are the maximum and minimum value of the qth $(q = 1,2,\dots,k)$ crisp objective function of (P3) obtained by solving it individually with the constraints of (P3) respectively.

Let $U_q(\delta)$ and $L_q(\delta)$ be the ideal and anti-ideal optimal values of the qth ($q = 1, 2, \ldots, k$) crisp objective function of (P3) respectively. It is assume that $L_q(\delta) \neq U_q(\delta)$, for all $q = 1, 2, \ldots, k$.

Step 2(b): Define the linear membership function corresponding to qth ($q = 1, 2, \ldots, k$) crisp objective function of (P3) as follow:

$$\mu_q(\mathrm{Mag}(Z_q(Y))) = \begin{cases} \dfrac{(\mathrm{Mag}(Z_q(Y)) - L_q(\delta))}{U_q(\delta) - L_q(\delta)}, & \text{if } L_q(\delta) \leq \mathrm{Mag}(Z_q(Y)) \leq U_q(\delta), \\ 0, & \text{otherwise,} \end{cases}$$

$$q = 1, 2, \ldots, k.$$

Step 3: Using goal programming (GP) technique, (P3) can be written as:

$$\text{(P4)} \quad \mathrm{Min} = \sum_{q=1}^{k} d_q^-$$

subject to

$$\mu_q(\mathrm{Mag}(Z_q(Y))) + d_q^- - d_q^+ = 1, \quad q = 1, 2, \ldots, k,$$

$$s\left[\sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}) \oplus \tilde{s}_i, \tilde{P}_i(z_i)\right] \geq \delta_i, \quad i = 1, 2, \ldots, m_1,$$

$$s\left[\sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}) \ominus \tilde{s}_i, \tilde{P}_i(z_i)\right] \geq \delta_i, \quad i = m_1 + 1, m_1 + 2, \ldots, m_2,$$

$$s\left[\sum_{j=1}^{n} \tilde{Q}_{ij}(v_j^{(i)}), \tilde{P}_i(z_i)\right] \geq \delta_i, \quad i = m_2 + 1, m_2 + 2, \ldots, m,$$

$$z_i = 0, 1, \ldots, (k_i - 1), \quad i = 1, 2 \ldots, m,$$

$$u_j^{(q)} = 0, 1, \ldots, (t_j^{(q)} - 1), \quad j = 1, 2, \ldots, n, \; q = 1, 2, \ldots, k,$$

$$v_j^{(i)} = 0, 1, \ldots, (s_j^{(i)} - 1), \quad j = 1, 2, \ldots, n, \; i = 1, 2 \ldots, m,$$

$$d_q^+ d_q^- = 0, \quad q = 1, 2, \ldots, k,$$

$$d_q^+, d_q^- \geq 0, \quad q = 1, 2, \ldots, k,$$

$$x_{j1}, (x_{j2} - x_{j1}), (x_{j3} - x_{j2}), (x_{j4} - x_{j3}) \geq 0, \quad j = 1, 2, \ldots, n,$$

$$s_{p1}, (s_{p2} - s_{p1}), (s_{p3} - s_{p2}), (s_{p4} - s_{p3}) \geq 0, \quad p = 1, 2, \ldots, m_2,$$

where d_q^- and d_q^+ are the negative and positive deviation of qth membership function respectively.

Step 4: Solve (P4) by LINGO 14.0 and find the optimal solution.

The FFMMOLP problem (P1) is transformed into (P4) using the similarity measure and magnitude of TrFN. The main feature of (P4) is that the constraints and objective functions are crisp by introducing the magnitude of fuzzy number and similarity measure $\delta, \delta = (\delta_1, \delta_2, \ldots, \delta_m)$ where δ_i $(i = 1, 2, \ldots, m)$ is the similarity measure of ith constraint specified by DM. So (P4) is called δ-parametric CNLP problem. Now, we define the δ-feasible solution of (P4) as follows:

Definition 4.3. The $(x_{jr}, s_{pr}, z_i, u_j^{(q)}, v_j^{(i)}, d_q^+, d_q^-)(\delta)$, $(r = 1, 2, 3, 4, p = 1, 2, \ldots, m_2,$ $j = 1, 2, \ldots, n,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ is called a δ-feasible solution of (P4) within the similarity measure δ, $\delta = (\delta_1, \delta_2 \ldots \delta_m)$ if it satisfies the constraints of (P4).

Let $X(\delta)$ denote the set of all δ-feasible solutions of (P4).

Definition 4.4. The solution $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_j^{(i)*}, d_q^{+*}, d_q^{-*})(\delta) \in X(\delta)$ is the δ-optimal solution of (P4) if

$$\sum_{q=1}^{k} d_q^{-*} \leq \sum_{q=1}^{k} d_q^- \quad \text{for all } (x_{jr}, s_{pr}, z_i, u_j^{(q)}, v_j^{(i)}, d_q^+, d_q^-)(\delta) \in X(\delta).$$

Remark 4.1. The optimal solution obtained in Step 4 is δ-optimal solution of (P4).

Now, we are going to show that the δ-optimal solution of (P4) gives the fuzzy Pareto-optimal solution of (P1).

Theorem 4.1. *Let* $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_j^{(i)*}, d_q^{+*}, d_q^{-*})(\delta) \in X(\delta)$, $(r = 1, 2, 3, 4,$ $p = 1, 2, \ldots, m_2,$ $j = 1, 2, \ldots n,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ *be the δ-optimal solution of (P4) then* $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$, *where* $\tilde{x}_j^* = (x_{j1}^*, x_{j2}^*, x_{j3}^*, x_{j4}^*)$, $(j = 1, 2 \ldots n)$ *will be the fuzzy Pareto-optimal solution of (P1).*

Proof. Let, if possible, $\tilde{X}^*$ be not fuzzy Pareto-optimal solution of (P1), then there exist a feasible solution $\tilde{X}^\circ \in F(\tilde{X})$, where $\tilde{X}^\circ = [\tilde{x}_j^\circ]_{n\times 1}$, $\tilde{x}_j^\circ = (x_{j1}^\circ, x_{j2}^\circ, x_{j3}^\circ, x_{j4}^\circ)$ $(j = 1, 2, \ldots, n)$ of (P1) such that

$$\left.\begin{array}{ll} \mathrm{Mag}(Z_q(\tilde{X}^\circ)) \geq \mathrm{Mag}(Z_q(\tilde{X}^*)) & \text{for all } q = 1, 2, \ldots, k \\ \qquad\qquad\text{and} & \\ \mathrm{Mag}(Z_w(\tilde{X}^\circ)) > \mathrm{Mag}(Z_w(\tilde{X}^*)) & \text{for at least one } w = 1, 2, \ldots, k \end{array}\right\} \tag{4.1}$$

Since (P2) is equivalent to (P1), therefore corresponding to the feasible solution $\tilde{X}^\circ$ of (P1), there exist $\tilde{S}^\circ = [\tilde{s}_p^\circ]_{m_2\times 1}$, where $\tilde{s}_p^\circ = (s_p^\circ, s_p^\circ, s_p^\circ, s_p^\circ)$, z_i°, $u_j^{(q)\circ}$ and $v_j^{(i)\circ}$ $(p = 1, 2, 3, 4,$ $j = 1, 2, \ldots, n,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ such that $(\tilde{X}^\circ, \tilde{S}^\circ, z_i^\circ, u_j^{(q)\circ}, v_j^{(i)\circ})$, $(j = 1, 2, \ldots, n,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ become the feasible solution of (P2).

Using Definition 3.1, it can be easily observe that $(x_{jr}^\circ, s_{ip}^\circ, z_i^\circ, u_j^{(q)\circ}, v_j^{(i)\circ})(\delta)$ $(r = 1, 2, 3, 4,$ $j = 1, 2, \ldots, n,$ $p = 1, 2, \ldots, m_2,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ is the feasible solution of (P3) corresponding to the feasible solution $(\tilde{X}^\circ, \tilde{S}^\circ, z_i^\circ, u_j^{(q)\circ}, v_j^{(i)\circ})$ $(j = 1, 2, \ldots, n,$ $q = 1, 2, \ldots, k,$ $i = 1, 2, \ldots, m)$ of (P2).

Denote $(x_{jr}^\circ, s_{ip}^\circ, z_i^\circ, u_j^{(q)\circ}, v_j^{(i)\circ})(\delta)$ by $Y^\circ(\delta)$.

Now $Y^\circ(\delta)$ is also feasible solution of all the single objective CNLP problems corresponding to the qth crisp objective function of (P3). $U_q(\delta)$ and $L_q(\delta)$ are ideal and anti-ideal values of qth crisp objective function of (P3) respectively. Therefore, we have

$$U_q(\delta) \geq \text{Mag}(Z_q(Y^{\circ})(\delta)) \geq L_q(\delta) \tag{4.2}$$

Let $\dfrac{(\text{Mag}(Z_q(Y^{\circ})(\delta)) - L_q(\delta))}{(U_q(\delta) - L_q(\delta))} = D_q^{\circ}$ for all $q = 1, 2, \ldots, k$.

Then $1 \geq D_q^{\circ} \geq 0$ and using constraints of (P4), we can observe that there exist $d_q^{+\circ} = 0$ and $0 \leq d_q^{-\circ} \leq 1$ $(q = 1, 2, \ldots, k)$ such that
$(\tilde{X}^{\circ}, \tilde{S}^{\circ}, z_i^{\circ}(\delta), u_j^{(q)\circ}, v_j^{(i)\circ}, d_q^{+\circ}, d_q^{-\circ})(\delta)$ is the feasible solution of (P4).

Also $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_i^{(q)*}, d_q^{+*}, d_q^{-*})(\delta)$ is the feasible solution of all CNLP problems corresponding to the qth crisp objective function of (P3).

Denote $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_j^{(i)*})(\delta)$ by $Y^*(\delta)$.

Hence

$$U_q(\delta) \geq \text{Mag}(Z_q(Y^*)(\delta)) \geq L_q(\delta) \tag{4.3}$$

$(x_{jr}^{\circ}, s_{ip}^{\circ}, \text{z}_i^{\circ}, u_j^{(q)\circ}, v_j^{(i)\circ}, d_q^{+\circ}, d_q^{-\circ})(\delta)$, $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_j^{(i)*}, d_q^{+*}, d_q^{-*})(\delta) \in X(\delta)$ both are feasible solution of (P4).

From (4.1), (4.2) and (4.3) we get

$$U_q(\delta) \geq \text{Mag}(Z_q(Y^{\circ})(\delta)) \geq \text{Mag}(Z_q(Y^*)(\delta)) \geq L_q(\delta) \text{ for all } q = 1, 2, \ldots, k$$

and

$$U_w(\delta) \geq \text{Mag}(Z_w(Y^{\circ})(\delta)) > \text{Mag}(Z_w(Y^*)(\delta)) \geq L_w(\delta) \text{ for at least one } w = 1, 2, \ldots, k$$

Let $\dfrac{(\text{Mag}(Z_q(Y^*)(\delta)) - L_q(\delta))}{(U_q(\delta) - L_q(\delta))} = D_q^*$ for all $q = 1, 2, \ldots, k$.

$$\Rightarrow \left.\begin{array}{l} D_q^{\circ} \geq D_q^* \text{ for all } q = 1, 2, \ldots, k \\ \quad \text{and} \\ D_w^{\circ} > D_w^* \text{ for at least one } w = 1, 2, \ldots, k \end{array}\right\}$$

$$\Rightarrow \left.\begin{array}{l} D_q^{\circ} - 1 \geq D_q^* - 1 \text{ for all } q = 1, 2, \ldots, k \\ \qquad \text{and} \\ D_w^{\circ} - 1 > D_w^* - 1 \text{ for at least one } w = 1, 2, \ldots, k \end{array}\right\}$$

Using the constraints of (P4), we get

$$\left.\begin{array}{l} d_q^{\circ+} - d_q^{\circ-} \geq d_q^{*+} - d_q^{*-} \text{ for all } q = 1, 2, \ldots, k \\ \qquad \text{and} \\ d_w^{\circ+} - d_w^{\circ-} > d_w^{*+} - d_w^{*-} \text{ for at least one } w = 1, 2, \ldots, k. \end{array}\right\}$$

Since $d_q^{\circ+}, d_q^{*+} = 0$ for all $q = 1, 2, \ldots, k$.

$$\Rightarrow \left.\begin{array}{l} d_q^{\circ-} \leq d_q^{*-} \text{ for all } q = 1, 2, \ldots, k \\ \qquad \text{and} \\ d_w^{\circ-} < d_w^{*-} \text{ for at least one } w = 1, 2, \ldots, k. \end{array}\right\} \tag{4.4}$$

From (4.4) we get $\sum\limits_{q=1}^{k} d_q^{\circ -} < \sum\limits_{q=1}^{k} d_q^{*-}$.

This is a contradiction as $(x_{jr}^*, s_{pr}^*, z_i^*, u_j^{(q)*}, v_j^{(i)*}, d_q^{+*}, d_q^{-*})(\delta)$ $(r = 1,2,3,4,\ j = 1,2,\dots,n,\ p = 1,2,\dots,m_2,\ q = 1,2,\dots,k,\ i = 1,2,\dots,m)$ is the optimal solution of (P4).

Hence, $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$, where $\tilde{x}_j^* = (x_{j1}^*, x_{j2}^*, x_{j3}^*, x_{j4}^*)$ $(j = 1,2,\dots,n)$ will be fuzzy Pareto-optimal solution of (P1). □

5. Example

In this section, the proposed method is illustrated through the following example. The same example is solved with the help of Hiseh and Chen [17] and Lee[25] similarity measure and comparison is made.

$$
\left.
\begin{aligned}
&\max Z_1(\tilde{X}) = ((5,6,7,8) \text{ or } (4,5,7,8))\otimes\tilde{x}_1 \oplus (4,5,6,7)\otimes\tilde{x}_2 \oplus (1,2,3,4)\otimes\tilde{x}_3\\
&\max Z_2(\tilde{X}) = (6,7,8,9)\otimes\tilde{x}_1 \oplus ((5,6,7,8) \text{ or } (5,6,8,9))\otimes\tilde{x}_2 \oplus (2,3,4,5)\otimes\tilde{x}_3\\
&\max Z_3(\tilde{X}) = (3,4,6,7)\otimes\tilde{x}_1 \oplus (2,3,4,5)\otimes\tilde{x}_2 \oplus ((2,4,5,6) \text{ or } (1,3,4,5))\otimes\tilde{x}_3\\
&\text{subject to} \quad ((2,4,5,6) \text{ or } (2,5,6,7))\otimes\tilde{x}_1 \oplus (2,3,4,5)\otimes\tilde{x}_2 \oplus (1,2,3,4)\otimes\tilde{x}_3\\
&\qquad\qquad \cong (8,16,20,28) \text{ or } (9,17,21,28) \text{ or } (8,17,20,29)\\
&\qquad\qquad (-2,-1,1,2)\otimes\tilde{x}_1 \oplus ((1,2,3,4) \text{ or } (2,4,5,6))\otimes\tilde{x}_2 \oplus (1,2,3,5)\otimes\tilde{x}_3\\
&\qquad\qquad \preceq (7,15,20,29) \text{ or } (6,13,17,24) \text{ or } (8,16,20,25)\\
&\qquad\qquad (2,3,4,5)\otimes\tilde{x}_1 \oplus (1,2,3,5)\otimes\tilde{x}_2 \oplus ((1,2,4,5) \text{ or } (2,3,4,5))\otimes\tilde{x}_3\\
&\qquad\qquad \succeq (10,18,22,30) \text{ or } (11,18,23,29) \text{ or } (10,18,21,29)\\
&\qquad\qquad \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \in \text{TrFN}(R)^+,
\end{aligned}
\right\}
\tag{5.1}
$$

where $\tilde{X} = [\tilde{x}_j]_{3\times 1}$, $j = 1,2,3$.

Using Definition 2.5, fuzzy Lagrange interpolating polynomial and adding or subtracting fuzzy slack or surplus variable to convert fuzzy inequality constraints into equality constraints, the problem (5.1) becomes:

Let $(\tilde{X}, \tilde{S}, \lambda_i, \beta_j) = Y'$.

$$
\left.
\begin{aligned}
&\max Z_1(Y') = ((5\beta_1+4(1-\beta_1))x_{11}+4x_{21}+x_{31}, (6\beta_1+5(1-\beta_1))x_{12}+5x_{22}+2x_{32},\\
&\qquad (7\beta_1+7(1-\beta_1))x_{13}+6x_{23}+3x_{33}, (8\beta_1+8(1-\beta_1))x_{14}+7x_{24}+4x_{34})\\
&\max Z_2(Y') = (6x_{11}+(5\beta_2+5(1-\beta_2))x_{21}+2x_{31}, 7x_{12}+(6\beta_2+6(1-\beta_2))x_{22}+3x_{32},\\
&\qquad 8x_{13}+(7\beta_2+8(1-\beta_2))x_{23}+4x_{33}, 9x_{14}+(8\beta_2+9(1-\beta_2))x_{24}+5x_{34})\\
&\max Z_3(Y') = (3x_{11}+2x_{21}+(2\beta_3+1(1-\beta_3))x_{31}, 4x_{12}+3x_{22}+(4\beta_3+3(1-\beta_3))x_{32},\\
&\qquad 6x_{13}+4x_{23}+(5\beta_3+4(1-\beta_3))x_{33}, 7x_{14}+5x_{24}+(6\beta_3+5(1-\beta_3))x_{34})\\
&\text{subject to}\\
&((2\lambda_1+2(1-\lambda_1))x_{11}+2x_{21}+x_{31}, (4\lambda_1+5(1-\lambda_1))x_{12}+3x_{22}+2x_{32},\\
&(5\lambda_1+6(1-\lambda_1))x_{13}+4x_{23}+3x_{33}, (6\lambda_1+7(1-\lambda_1))x_{14}+5x_{24}+4x_{34})\\
&\quad \cong \left(\tfrac{(z_1-1)(z_1-2)}{2}(8,16,20,28) - \tfrac{z_1(z_1-2)}{1}(9,17,21,28) + \tfrac{z_1(z_1-1)}{2}(8,17,20,29)\right)\\
&(-2x_{14}+(\lambda_2+2(1-\lambda_2))x_{21}+x_{31}+s_{11}, -x_{13}+(2\lambda_2+4(1-\lambda_2))x_{22}+2x_{32}+s_{12},\\
&x_{13}+(3\lambda_2+5(1-\lambda_2))x_{23}+3x_{33}+s_{13}, 2x_{14}+(4\lambda_2+6(1-\lambda_2))x_{24}+5x_{34}+s_{14})\\
&\quad \cong \left(\tfrac{(z_2-1)(z_2-2)}{2}(7,15,20,29) - \tfrac{z_2(z_2-2)}{1}(6,13,17,24) + \tfrac{z_2(z_2-1)}{2}(8,16,20,25)\right)\\
&(2x_{11}+x_{21}+(\lambda_3+2(1-\lambda_3))x_{31}-s_{24}, 3x_{12}+2x_{22}+(2\lambda_3+3(1-\lambda_3))x_{32}-s_{23},\\
&4x_{13}+3x_{23}+(4\lambda_3+4(1-\lambda_3))x_{33}-s_{22}, 5x_{14}+5x_{24}+(5\lambda_3+5(1-\lambda_3))x_{34}-s_{21})\\
&\quad \cong \left(\tfrac{(z_3-1)(z_3-2)}{2}(10,18,22,30) - \tfrac{z_3(z_3-2)}{1}(11,18,23,29) + \tfrac{z_3(z_3-1)}{2}(10,18,21,29)\right)\\
&z_i = 0,1,2 \ \ \forall\, i = 1,2,3, \quad \lambda_1,\lambda_2,\lambda_3,\beta_1,\beta_2,\beta_3 = 0 \text{ or } 1, \quad \tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{s}_1,\tilde{s}_2 \in \text{TrFN}(R)^+,
\end{aligned}
\right\}
\tag{5.2}
$$

where $\tilde{S} = [\tilde{s}_p]_{2\times 1}$, $(p = 1,2$ and $i,j = 1,2,3)$.

Step 1: Using Definition 3.1 and Definition 2.6, (5.2) is converted into CMONLP problem as follows:

$$\left.\begin{aligned}
&\max[\mathrm{Mag}(Z_1(Y'))] = \frac{C_1}{12}\\
&\max[\mathrm{Mag}(Z_2(Y'))] = \frac{C_2}{12}\\
&\max[\mathrm{Mag}(Z_3(Y'))] = \frac{C_3}{12}\\
&\text{subject to}\\
&\quad 48 \geq |A_{11}+A_{12}+A_{13}+A_{14}|\\
&\quad 48 \geq |B_{11}+B_{12}+B_{13}+B_{14}|\\
&\quad 48 \geq |C_{11}+C_{12}+C_{13}+C_{14}|\\
&\quad z_i = 0,1,2 \quad \forall\ i=1,2,3,\\
&\quad \lambda_1,\lambda_2,\lambda_3,\beta_1,\beta_2,\beta_3 = 0 \text{ or } 1,\\
&\quad x_{j1}, x_{j2}-x_{j1}, x_{j3}-x_{j2}, x_{j4}-x_{j3} \geq 0 \ \ \forall\ j=1,2,3,\\
&\quad s_{p1}, s_{p2}-s_{p1}, s_{p3}-s_{p2}, s_{p4}-s_{p3} \geq 0 \ \ \forall\ p=1,2,
\end{aligned}\right\} \tag{5.3}$$

where

$$\begin{aligned}
C_1 = {}&(5\beta_1+4(1-\beta_1))x_{11}+4x_{21}+x_{31}+(30\beta_1+25(1-\beta_1))x_{12}+25x_{22}+10x_{32}\\
&+(35\beta_1+35(1-\beta_1))x_{13}+30x_{23}+15x_{33}+(8\beta_1+8(1-\beta_1))x_{14}+7x_{24}+4x_{34})
\end{aligned}$$

$$\begin{aligned}
C_2 = {}&(6x_{11}+(5\beta_2+5(1-\beta_2))x_{21}+2x_{31}+35x_{12}+(30\beta_2+30(1-\beta_2))x_{22}+15x_{32}\\
&+40x_{13}+(35\beta_2+40(1-\beta_2))x_{23}+20x_{33}+9x_{14}+(8\beta_2+9(1-\beta_2))x_{24}+5x_{34})
\end{aligned}$$

$$\begin{aligned}
C_3 = {}&(3x_{11}+2x_{21}+(2\beta_3+(1-\beta_3))x_{31}+20x_{12}+15x_{22}+(20\beta_3+15(1-\beta_3))x_{32}\\
&+30x_{13}+20x_{23}+(25\beta_3+20(1-\beta_3))x_{33}+7x_{14}+5x_{24}+(6\beta_3+5(1-\beta_3))x_{34})
\end{aligned}$$

$$A_{11} = (2\lambda_1+2(1-\lambda_1))x_{11}+2x_{21}+x_{31}-\tfrac{(z_1-1)(z_1-2)}{2}8+\tfrac{z_1(z_1-2)}{1}9-\tfrac{z_1(z_1-1)}{2}8$$

$$A_{12} = (20\lambda_1+25(1-\lambda_1))x_{12}+15x_{22}+10x_{32}-\tfrac{(z_1-1)(z_1-2)}{2}80+\tfrac{z_1(z_1-2)}{1}85-\tfrac{z_1(z_1-1)}{2}85$$

$$A_{13} = (25\lambda_1+30(1-\lambda_1))x_{13}+20x_{23}+15x_{33}-\tfrac{(z_1-1)(z_1-2)}{2}100+\tfrac{z_1(z_1-2)}{1}105-\tfrac{z_1(z_1-1)}{2}100$$

$$A_{14} = (6\lambda_1+7(1-\lambda_1))x_{14}+5x_{24}+4x_{34}-\tfrac{(z_1-1)(z_1-2)}{2}28+\tfrac{z_1(z_1-2)}{1}28-\tfrac{z_1(z_1-1)}{2}29$$

$$B_{11} = -2x_{14}+(\lambda_2+2(1-\lambda_2))x_{21}+x_{31}+s_{11}-\tfrac{(z_2-1)(z_2-2)}{2}7+\tfrac{z_2(z_2-2)}{1}6-\tfrac{z_2(z_2-1)}{2}8$$

$$B_{12} = -5x_{13}+(10\lambda_2+20(1-\lambda_2))x_{22}+10x_{32}+5s_{12}-\tfrac{(z_2-1)(z_2-2)}{2}75+\tfrac{z_2(z_2-2)}{1}65-\tfrac{z_2(z_2-1)}{2}80$$

$$\begin{aligned}
B_{13} = {}&5x_{13}+(15\lambda_2+25(1-\lambda_2))x_{23}+15x_{33}+5s_{13}-\tfrac{(z_2-1)(z_2-2)}{2}100\\
&+\tfrac{z_2(z_2-2)}{1}85-\tfrac{z_2(z_2-1)}{2}100
\end{aligned}$$

$$B_{14} = 2x_{14} + (4\lambda_2 + 6(1-\lambda_2))x_{24} + 5x_{34} + s_{14} - \tfrac{(z_2-1)(z_2-2)}{2}29 + \tfrac{z_2(z_2-2)}{1}24 - \tfrac{z_2(z_2-1)}{2}25$$

$$C_{11} = 2x_{11} + x_{21} + (\lambda_3 + 2(1-\lambda_3))x_{31} - s_{24} - \tfrac{(z_3-1)(z_3-2)}{2}10 + \tfrac{z_3(z_3-2)}{1}11 - \tfrac{z_3(z_3-1)}{2}10$$

$$C_{12} = 15x_{12} + 10x_{22} + (10\lambda_3 + 15(1-\lambda_3))x_{32} - 5s_{23} - \tfrac{(z_3-1)(z_3-2)}{2}90 + \tfrac{z_3(z_3-2)}{1}90 - \tfrac{z_3(z_3-1)}{2}90$$

$$C_{13} = 20x_{13} + 10x_{23} + (20\lambda_3 + 20(1-\lambda_3))x_{33} - 5s_{22} - \tfrac{(z_3-1)(z_3-2)}{2}110 + \tfrac{z_3(z_3-2)}{1}115 - \tfrac{z_3(z_3-1)}{2}105$$

$$C_{14} = 5x_{14} + 5x_{24} + (5\lambda_3 + 5(1-\lambda_3))x_{34} - s_{21} - \tfrac{(z_3-1)(z_3-2)}{2}30 + \tfrac{z_3(z_3-2)}{1}29 - \tfrac{z_3(z_3-1)}{2}29$$

Step 2: Define the linear membership function for the qth, $(q = 1, 2, 3)$ crisp objective function of (5.3) as follows:

Step 2(a): The ideal and anti-ideal values of the qth $(q = 1, 2, 3)$ crisp objective function of (5.3) are listed below in Table 1.

Table 1.

Crisp objective function	**Ideal value**	**Anti-ideal value**
$\text{Mag}(Z_1(Y'))$	35.42	14.5
$\text{Mag}(Z_2(Y'))$	45.14	18.12
$\text{Mag}(Z_3(Y'))$	39.42	15.42

Step 2(b): Linear membership function corresponding to qth $(q = 1, 2, 3)$ crisp objective function of (5.3) is defined as follow:

$$\mu_1(\text{Mag}(Z_1(Y'))) = \begin{cases} \dfrac{(\text{Mag}(Z_1(Y')) - 14.5)}{35.42 - 14.5} & \text{if } 14.5 \le \text{Mag}(Z_q(Y')) \le 35.42, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_2(\text{Mag}(Z_2(Y'))) = \begin{cases} \dfrac{(\text{Mag}(Z_2(Y')) - 18.12)}{41.67 - 18.12} & \text{if } 18.12 \le \text{Mag}(Z_q(Y')) \le 41.67, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_3(\text{Mag}(Z_3(Y'))) = \begin{cases} \dfrac{(\text{Mag}(Z_3(Y')) - 15.42)}{40.48 - 15.42} & \text{if } 15.42 \le \text{Mag}(Z_q(Y')) \le 40.48, \\ 0 & \text{otherwise.} \end{cases}$$

Step 3: By GP approach, (CMONLP) problem can be written as:

$$
(5.4)\quad \left.\begin{array}{l}
\text{Min} = d_1^- + d_2^- + d_3^- \\
\text{subject to} \\
\quad \mu_i(\text{Mag}(Z_i(Y'))) + (d_i^- - d_i^+) = 1 \ \ \forall\, i = 1,2,3 \\
\quad 48 \geq |A_{11} + A_{12} + A_{13} + A_{14}| \\
\quad 48 \geq |B_{11} + B_{12} + B_{13} + B_{14}| \\
\quad 48 \geq |C_{11} + C_{12} + C_{13} + C_{14}| \\
\quad z_i = 0,1,2 \ \ \forall\, i = 1,2,3, \\
\quad \lambda_1, \lambda_2, \lambda_3, \beta_1, \beta_2, \beta_3 = 0 \text{ or } 1, \\
\quad x_{j1}, x_{j2} - x_{j1}, x_{j3} - x_{j2}, x_{j4} - x_{j3} \geq 0 \ \ \forall\, j = 1,2,3, \\
\quad s_{p1}, s_{p2} - s_{p1}, s_{p3} - s_{p2}, s_{p4} - s_{p3} \geq 0 \ \ \forall\, p = 1,2.
\end{array}\right\}
$$

Solving (5.4) by LINGO 14.0, using Step 5 and Theorem 4.1, fuzzy Pareto-optimal solution and fuzzy optimal value of the qth ($q = 1,2,3$) objective functions of (5.1) are given below in Table 2 and Table 3, respectively.

TABLE 2.

Fuzzy Variables of (5.1)	**Fuzzy Pareto-optimal solution of (5.1)**
$\tilde{x}_1^*$	$(0,0,0,0)$
$\tilde{x}_2^*$	$(6.07,6.07,6.07,6.07)$
$\tilde{x}_3^*$	$(0.67,0.67,0.67,0.67)$

TABLE 3.

Fuzzy objective function of (5.1)	**Fuzzy objective function value of (5.1)**
$Z_1(\tilde{X}^*)$	$(24.95,31.69,38.43,45.17)$
$Z_2(\tilde{X}^*)$	$(31.69,38.43,51.24,57.98)$
$Z_2(\tilde{X}^*)$	$(12.81,20.89,27.63,34.37)$

The values of fuzzy Pareto-optimal solution and fuzzy objective functions obtained using the Hiseh and Chen[17] similarity measure and Lee[25] similarity measure of (5.1) are given below in Table 4 - Table 7 respectively.

TABLE 4.

Fuzzy Variables of (5.1)	**Fuzzy Pareto-optimal solution of (5.1) using Hiseh and Chen [17] similarity measure**
$\tilde{x}_1^*$	$(0.89,0.89,0.89,0.89)$
$\tilde{x}_2^*$	$(1.23,1.23,1.23,1.23)$
$\tilde{x}_3^*$	$(3.44,3.44,3.44,3.44)$

Table 5.

Fuzzy objective function of (5.1)	**Fuzzy objective function value of** (5.1) **using Hiseh and Chen** [17] **similarity measure**
$Z_1(\tilde{X}^*)$	$(12.81, 18.37, 23.04, 29.49)$
$Z_2(\tilde{X}^*)$	$(18.37, 23.93, 29.49, 35.05)$
$Z_2(\tilde{X}^*)$	$(12.01, 21.01, 27.46, 33.02)$

Table 6.

Fuzzy Variables of (5.1)	**Fuzzy Pareto-optimal solution of** (5.1) **using Lee** [25] **similarity measure**
$\tilde{x}_1^*$	$(1.08, 1.08, 1.08, 1.08)$
$\tilde{x}_2^*$	$(0.42, 0.42, 0.42, 0.42)$
$\tilde{x}_3^*$	$(2.88, 2.88, 2.98, 2.98)$

Table 7.

Fuzzy objective function of (5.1)	**Fuzzy objective function value of** (5.1) **using Lee** [25] **similarity measure**
$Z_1(\tilde{X}^*)$	$(9.96, 12.18, 19.02, 23.5)$
$Z_2(\tilde{X}^*)$	$(14.54, 18.72, 23.5, 27.98)$
$Z_2(\tilde{X}^*)$	$(10.04, 17.5, 23.04, 27.54)$

(5.1) is solved using Hiseh and Chen [17] similarity measure, Lee [25] similarity measure and proposed similarity measure. Hence, a comparison is made with the help of definition 2.7 in Table 8 as follows:

Table 8.

Using Lee [25] **similarity measure**		**Using Hiseh and Chen** [17] **similarity measure**		**Using proposed similarity measure**
$Z_1(\tilde{X}^*) = 15.78$	$<$	$Z_1(\tilde{X}^*) = 20.77$	$<$	$Z_1(\tilde{X}^*) = 35.06$
$Z_2(\tilde{X}^*) = 21.13$	$<$	$Z_2(\tilde{X}^*) = 26.70$	$<$	$Z_2(\tilde{X}^*) = 40.00$
$Z_3(\tilde{X}^*) = 20.02$	$<$	$Z_3(\tilde{X}^*) = 23.94$	$<$	$Z_3(\tilde{X}^*) = 24.14$

Thus, we can easily conclude from Table 8 that proposed similarity measure gives better fuzzy objective function values of (5.1) than Hiseh and Chen [17] and Lee [25] similarity measure.

6. Conclusion

In this paper, a new similarity measure is defined for fuzzy numbers and an explanation is given as to why some of the existing similarity measures cannot be applied to FFMMOLP problem. A novel approach for solving the FFMMOLP problem is proposed. In this method, we have applied similarity measure on the fuzzy constraints to convert them into crisp and magnitude on objective functions. Using this method fuzzy Pareto-optimal solution of FFMMOLP problem is obtained and with the help of numerical example, it is shown that the proposed similarity when applied on the FFMMOLP problem give better solution than Hsieh and Chen's [17] and Lee's [25] similarity measure. The classical fuzzy multi-objective, FFLP and multi-choice linear programming problems are special case of FFMMOLP problem. The major advantage of this model is that it gives more choices to the DM and DM can handle the unexpected problem that one has to face while making decision in the real world.

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Shashi Aggarwal (shashi.aggarwal@mirandahouse.ac.in)
Department of Mathematics, Miranda House, University of Delhi, Delhi-110007, India

UDAY SHARMA (udaysharma88@yahoo.com)
Department of Mathematics, University of Delhi, Delhi-110007, India