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A new approach for ranking of exponential fuzzy number with use optimism

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Abstract. The main aim of this paper is to propose a new approach for the ranking of generalized exponential trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. Also, with the help of several counter examples it is proved that ranking method proposed by Chen and Chen 2009 is incorrect.

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1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [1] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [2]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [3] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [4] reviewed some of these ranking methods for ranking fuzzy subsets and fuzzy numbers. Chen [5] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [6] presented the mean value of a fuzzy number. Chu and Tsao [7] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [8] presented a

centroid-index method for ranking fuzzy numbers. Liang et al. [9] and Wang and Lee [10] also used the centroid concept in developing their ranking index. Chen and Chen [11] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [12] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Chen and Chen [13] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Also Some of the interesting Approach Ranking Of Trapezoidal Fuzzy Number can be found in Amit Kumar [14]. Moreover, S, Rezvani [15-24] proposed a method for ranking in fuzzy numbers.
The main aim of this paper is to propose a new approach for the ranking of generalized exponential trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. Also, with the help of several counter examples it is proved that ranking method proposed by Chen 2009 is incorrect.

2. Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < u \leq c$,

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[c, a]$,

(iv) $\mu_A(x) = w, a \leq x \leq b$,

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[b, d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted bye $A = (c, a, b, d; w)_{LR}$.

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (c, a, b, d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c, d]$. Here, we define its general from as follows:

$$f_A(x) = \begin{cases} we^{-[(a-x)/\gamma]} & x \leq a, \\ w & a \leq x \leq b, \\ we^{-[(x-b)/\beta]} & b \leq x, \end{cases} \tag{2.1}$$

where $0 < w \leq 1$, a, b are real numbers, and γ, β are positive real numbers. we denote this type of generalized exponential fuzzy number as $A = (a, b, \gamma, \beta;\ w)_E$. Especially, when $w = 1$, we denote it as $A = (a, b, \gamma, \beta)_E$.
we define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a, b, \gamma, \beta)_E$, where $0 < w \leq 1$, and γ, β are positive real numbers, a, b are real numbers as in formula (2.1). Now, let two monotonic functions be

$$L(x) = we^{-[(a-x)/\gamma]},\ R(x) = we^{-[(x-b)/\beta]} \tag{2.2}$$

then the inverse functions of function L and R are L^{-1} and R^{-1} respectively. the h-level graded mean value of generalized exponential fuzzy number $A = (a, b, \gamma, \beta;\ w)_E$ can be express as

$$h[L^{-1}(h) + R^{-1}(h)]/2 \tag{2.3}$$

Definition 1. Let $A = (a, b, \gamma, \beta;\ w)_E$, be generalized exponential number, then the graded mean integration representation of A is define by

$$P(A) = (\int_0^w h(\frac{L^{-1}(h) + R^{-1}(h)}{2} dh) / \int_0^w h\ dh. \tag{2.4}$$

Theorem 1. Let $A = (a, b, \gamma, \beta;\ w)_E$, be generalized exponential number with $0 < w \leq 1$ and γ, β are positive real numbers, a, b are real numbers. then the graded mean integration representation of A is

$$P(A) = \frac{a+b}{2} + \frac{\beta - \gamma}{4}. \tag{2.5}$$

Proof:

$$L^{-1}(h) = a - \gamma(\ln \frac{w}{h}), \tag{2.6}$$

$$R^{-1}(h) = b + \beta(\ln \frac{w}{h}). \tag{2.7}$$

$$\begin{aligned}
P(A) &= \frac{1}{2} \int_0^w h[a + b + \beta(\ln \frac{w}{h}) - \gamma(\ln \frac{w}{h})] dh / \frac{1}{2} w^2 \\
&= \frac{a+b}{2} + \frac{\beta - \gamma}{2} \int_0^w h(\ln \frac{w}{h}) \\
&= \frac{a+b}{2} + \frac{\beta - \gamma}{2} [\int_0^w h \ln(w)\ dh - \int_0^w h \ln(h)\ dh] \\
&= \frac{a+b}{2} + \frac{\beta - \gamma}{2} \int_0^w h[\ln(w) - \ln(h)]\ dh \\
&= \frac{a+b}{2} + \frac{\beta - \gamma}{4}.
\end{aligned}$$

Remark 1. When $\gamma = \beta$, $P(A) = \frac{a+b}{2}$.

3. Arithmetic Operations of Exponential Fuzzy Numbers and Ranking Function

Definition 2. suppose that $A_1 = (a_1, b_1, \gamma_1, \beta_1;\ w_1)_E$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2;\ w_2)_E$ are two generalized exponential fuzzy numbers. Let $w = min\{w_1, w_2\}$, according to the essential of the second function principle, some arithmetical operations results could be well define as follows.

(i) The addition of A_1 and A_2 is

$$A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2;\ w)_E \tag{3.1}$$

where $\gamma_1, \gamma_2, a_1, a_2, b_1, b_2, \beta_1, \beta_2$ are all real numbers, and $\gamma_1, \gamma_2, \beta_1, \beta_2$ are positive.

(ii) The multiplication of A_1 and A_2 is

$$A_1 \otimes A_2 = (a, b, \gamma, \beta;\ w)_E. \tag{3.2}$$

Where $T = \{a_1a_2, a_1b_2, b_1a_2, b_1b_2\}$, $\quad T_1 = \{\gamma_1\gamma_2, \gamma_1\beta_2, \beta_1\gamma_2, \beta_1\beta_2\}$

and $a = min\ T = k^{th}$ element of T, and $b = max\ T = l^{th}$ element of T, then $\gamma = k^{th}$ element of T_1 and $\beta = l^{th}$ element of T_1, where $1 \leq k \leq 4, 1 \leq l \leq 4$.

(iii) $-A_2 = (-b_2, -a_2, \beta_2, \gamma_2; w)$, then

$$A_1 \ominus A_2 = A_1 \oplus (-A_2) = (a_1 - b_2, b_1 - a_2, \gamma_1 + \beta_2, \beta_1 + \gamma_2; w)_E. \tag{3.3}$$

(iv) Let $m \in R^+, A = (a, b, \gamma, \beta;\ w)_E$, then

$$m \otimes A = (ma, mb, m\gamma, m\beta;\ w)_E \tag{3.4}$$

if $m \in R^-, A = (a, b, \gamma, \beta;\ w)_E$, then

$$m \otimes A = (mb, ma, \mid m \mid \beta, \mid m \mid \gamma;\ w)_E \tag{3.5}$$

(v) $\frac{1}{A_2} = (\frac{1}{b_2}, \frac{1}{a_2}, \frac{1}{\beta_2}, \frac{1}{\gamma_2}; w)_E$, We have

$$\frac{A_1}{A_2} = A_1 \otimes (\frac{1}{A_2}) = (\frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{\gamma_1}{\beta_2}, \frac{\beta_1}{\gamma_2}; w)_E, \tag{3.6}$$

where if $a_1b_1, a_2b_2, \gamma_1, \gamma_2, \beta_1, \beta_2$ are all nonzero positive real numbers.

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function, $\Re :\ F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists,

(1) $A > B \ \ iff \ \ \Re(A) > \Re(B)$.

(2) $A < B \ \ iff \ \ \Re(A) < \Re(B)$.

(3) $A = B \ \ iff \ \ \Re(A) = \Re(B)$.

Remark 2. [15]. For all fuzzy numbers $A,\ B,\ C$ and D, we have

(1) $A > B \Rightarrow A \oplus C > B \oplus C$.

(2) $A > B \Rightarrow A \ominus\ C > B \ominus\ C$.

(3) $A \sim B \Rightarrow A \oplus C \sim B \oplus C$.

(4) $A > B, C > D \Rightarrow A \oplus C > B \oplus D$.

4. Shortcomings of Chen and Chen Approach [13]

In this section, on the basis of reasonable properties of fuzzy quantities [23] and on the basis of height of fuzzy numbers, are pointed out. Let A and B be any two fuzzy numbers. Then

$A > B \Rightarrow A \ominus B > B \ominus B$ (Using Remark 2.),
That is
$\Re(A) > \Re(B) \Rightarrow \Re(A \ominus B) > \Re(B \ominus B)$.

In this subsection, several examples are choosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property,

$$A > B \rightarrow A \ominus\ B > B \ominus\ B \tag{4.1}$$

for the ordering of fuzzy quantities i.e., according to Chen Chen approach

$A > B$, not result $A \ominus\ B > B \ominus\ B$

Example 1. Let $A = (0.1, 0.3, 0.3, 0.5; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$B \ominus A < A \ominus A$ *that is* , $B > A$ *not result* $B \ominus A > A \ominus A$.

Example 2. Let $A = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$,but

$B \ominus A < A \ominus A$ *that is* , $B > A$ *not result* $B \ominus A > A \ominus A$.

Example 3. Let $A = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $B = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A > B$, but

$A \ominus B < B \ominus B$ *that is* , $A > B$ *not result* $A \ominus B > B \ominus B$.

Example 4. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach

$B > A$, but

$B \ominus A < A \ominus A$ *that is* , $B > A$ *not result* $B \ominus A > A \ominus A$.

In some cases, Chen and Chen approach [2] states that the ranking of fuzzy numbers depends upon height of fuzzy numbers while in several cases the ranking does not depend upon the height of fuzzy numbers.
Let $A_1 = (a_1, b_1, \gamma_1, \beta_1;\ w_1)$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2;\ w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen [13] there may be two cases.

$Case(1)$ If $(a_1 + b_1 + \gamma_1 + \beta_1) \neq 0$, then

$$\begin{cases} A < B \quad if \quad w_1 < w_2 \\ A > B \quad if \quad w_1 > w_2 \\ A \sim B \quad if \quad w_1 \sim w_2. \end{cases} \tag{4.2}$$

$Case(2)$ If $(a_1 + b_1 + \gamma_1 + \beta_1) = 0$, then $A \sim B$ for all values of w_1 and w_2.

According to Chen and Chen [13] in first case ranking of fuzzy numbers depends upon height and in second case ranking does not depend upon the height which is contradiction.

Example 5. Let $A = (1, 1, 1, 1; w_1)$ and $B = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A < B$ if $w_1 < w_2$, $A > B$ if $w_1 > w_2$ and $A = B$ if $w_1 = w_2$.

Example 6. Let $A = (-0.4, -0.2, -0.1, 0.7; w_1)$ and $B = (-0.4, -0.2, -0.1, 0.7; w_2)$ be two generalized triangular fuzzy numbers. Then $A = B$ for all values of w_1 and w_2.

5. Proposed Approach

In this section, on the basis of property of ranking function, discussed in Section 3, a new approach is proposed for the ranking of generalized exponential trapezoidal fuzzy numbers.
Let $A_1 = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two generalized trapezoidal fuzzy numbers. Then

(1) $A > B$ if $RM(A \ominus B) > RM(B \ominus B)$,

(2) $A < B$ if $RM(A \ominus B) < RM(B \ominus B)$,

(3) $A = B$ if $RM(A \ominus B) = RM(B \ominus B)$.

5.1. Method to Find Value of $\Re(A)$ and $\Re(B)$.

Let $A = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $B = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two any generalized exponential trapezoidal fuzzy number Then with use [14], find the values of

$*$ step 1. Find $w = min\{w_1, w_2\}$

$*$ step 2. Let $\mu_A(x)$ and $\mu_B(x)$ are the membership function of A and B respectively, Then

$$\mu_A(x) = \begin{cases} w_1 L_1 e^{-[(a_1-x)/\gamma_1]} & x \leq a_1, \\ w_1 & a_1 \leq x \leq b_1, \\ w_1 R_1 e^{-[(x-b_1)/\beta_1]} & b_1 \leq x, \end{cases} \tag{5.1}$$

and

$$\mu_B(x) = \begin{cases} w_2 L_2 e^{-[(a_2-x)/\gamma_2]} & x \leq a_2, \\ w_2 & a_2 \leq x \leq b_2, \\ w_2 R_2 e^{-[(x-b_2)/\beta_2]} & b_2 \leq x, \end{cases} \tag{5.2}$$

Put $w = min\{w_1, w_2\}$, Then

$$\mu_A(x) = \begin{cases} w L_1 e^{-[(a_1-x)/\gamma_1]} & x \leq a_1, \\ w & a_1 \leq x \leq b_1, \\ w R_1 e^{-[(x_1-b)/\beta_1]} & b_1 \leq x, \end{cases} \tag{5.3}$$

and

$$\mu_B(x) = \begin{cases} w L_2 e^{-[(a_2-x)/\gamma_2]} & x \leq a_2, \\ w & a_2 \leq x \leq b_2, \\ w R_2 e^{-[(x-b_2)/\beta_2]} & b_2 \leq x, \end{cases} \tag{5.4}$$

With use equation (6), (7), we have

$$L_1(h) = wL_1 e^{-[(a_1-x)/\gamma_1]} \Rightarrow L_1^{-1}(h) = a_1 - \gamma_1(\ln \frac{w}{h}),$$

$$R_1(h) = wR_1 e^{-[(x-b_1)/\beta_1]} \Rightarrow R_1^{-1}(h) = b_1 + \beta_1(\ln \frac{w}{h}).$$

and

$$L_2(h) = wL_2 e^{-[(a_2-x)/\gamma_2]} \Rightarrow L_2^{-1}(h) = a_2 - \gamma_2(\ln \frac{w}{h}),$$

$$R_2(h) = wR_2 e^{-[(x-b_2)/\beta_2]} \Rightarrow R_2^{-1}(h) = b_2 + \beta_2(\ln \frac{w}{h}).$$

So

$$\Re(A) = \alpha \int_0^w [a_1 - \gamma_1(\ln \frac{w}{h})]\ dh + (1-\alpha) \int_0^w [b_1 + \beta_1(\ln \frac{w}{h})]\ dh$$

$$= \alpha[a_1 w - \int_0^w \gamma_1(\ln w - \ln h)\ dh] + (1-\alpha)[b_1 w - \int_0^w \beta_1(\ln w - \ln h)\ dh]$$

$$= \alpha[a_1 w - \gamma_1 w] + (1-\alpha)[b_1 w + \beta_1 w] = \alpha w[a_1 - \gamma_1] + (1-\alpha)w[b_1 + \beta_1]. \tag{5.5}$$

and

$$\Re(B) = \alpha \int_0^w [a_2 - \gamma_2(\ln \frac{w}{h})]\ dh + (1-\alpha) \int_0^w [b_2 + \beta_2(\ln \frac{w}{h})]\ dh$$

$$= \alpha[a_2 w - \int_0^w \gamma_2(\ln w - \ln h)\ dh] + (1-\alpha)[b_2 w - \int_0^w \beta_2(\ln w - \ln h)\ dh]$$

$$= \alpha[a_2w - \gamma_2w] + (1-\alpha)[b_2w + \beta_2w] = \alpha w[a_2 - \gamma_2] + (1-\alpha)w[b_2 + \beta_2]. \tag{5.6}$$

Therefore

$$\Re(A) = \alpha w[a_1 - \gamma_1] + (1-\alpha)w[b_1 + \beta_1],$$

$$\Re(B) = \alpha w[a_2 - \gamma_2] + (1-\alpha)w[b_2 + \beta_2].$$

Indeed, The index of optimism (α) is representing the degree of optimism of a decision maker [16]. A larger value of α indicates a higher degree of optimism. For $\alpha = 0$ and $\alpha = 1$ value of $\Re(A)$ and $\Re(B)$ represents the view points of a pessimistic and optimistic decision maker respectively while for $\alpha = 0.5$ values of $\Re(A)$ and $\Re(B)$ represents the view points of a moderate decision maker.

Remark 3. We get for generalized exponential trapezoidal fuzzy numbers

$$\Re(A) = \alpha w[a_1 - \gamma_1] + (1-\alpha)w[b_1 + \beta_1],$$

$$\Re(B) = \alpha w[a_2 - \gamma_2] + (1-\alpha)w[b_2 + \beta_2].$$

and normalize fuzzy numbers

$$\Re(A) = \alpha[a_1 - \gamma_1] + (1-\alpha)[b_1 + \beta_1],$$

$$\Re(B) = \alpha[a_2 - \gamma_2] + (1-\alpha)[b_2 + \beta_2].$$

Remark 4. The arithmetic operations between two fuzzy numbers is obtained using the α-cut method [13] and the maximum value of h, that will be common for both fuzzy numbers, will be obtained by finding the minimum value of the height of the fuzzy numbers, due to which in $w = min\{w_1, w_2\}$ is considered.

Theorem 2. Prove that ranking of generalized exponential fuzzy numbers does not depend upon the height of fuzzy numbers i.e., if A, B are two generalized exponential fuzzy numbers and C, D are normalize fuzzy numbers, obtained from A, B respectively, then

$$(i) \quad A > B \ \ iff \ \ C > D \ ,$$

$$(ii) \quad A < B \ \ iff \ \ C < D \ ,$$

$$(iii) \quad A \sim B \ \ iff \ \ C \sim D \ ,$$

Proof: Let $A = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $B = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two any generalized exponential trapezoidal fuzzy number and let $C = (a_1, b_1, \gamma_1, \beta_1; 1)$ and $D = (a_2, b_2, \gamma_2, \beta_2; 1)$ are normal exponential trapezoidal fuzzy number of A, B respectively and $w = min\{w_1, w_2\}$. Then

(i) Let $A > B \Leftrightarrow \Re(A) > \Re(B) \Leftrightarrow \alpha w[a_1 - \gamma_1] + (1-\alpha)w[b_1 + \beta_1] > \alpha w[a_2 - \gamma_2] + (1-\alpha)w[b_2 + \beta_2] \Leftrightarrow \alpha[a_1 - \gamma_1] + (1-\alpha)[b_1 + \beta_1] > \alpha[a_2 - \gamma_2] + (1-\alpha)[b_2 + \beta_2] \Leftrightarrow \Re(C) > \Re(D) \Leftrightarrow C > D$.

(ii) Let $A < B \Leftrightarrow \Re(A) < \Re(B) \Leftrightarrow \alpha w[a_1 - \gamma_1] + (1-\alpha)w[b_1 + \beta_1] < \alpha w[a_2 - \gamma_2] +$

$(1-\alpha)w[b_2+\beta_2] \Leftrightarrow \alpha[a_1-\gamma_1]+(1-\alpha)[b_1+\beta_1] < \alpha[a_2-\gamma_2]+(1-\alpha)[b_2+\beta_2] \Leftrightarrow \mathfrak{R}(C) < \mathfrak{R}(D) \Leftrightarrow C < D$.

(iii) Let $A \sim B \Leftrightarrow \mathfrak{R}(A) \sim \mathfrak{R}(B) \Leftrightarrow \alpha w[a_1-\gamma_1]+(1-\alpha)w[b_1+\beta_1] \sim \alpha w[a_2-\gamma_2]+(1-\alpha)w[b_2+\beta_2] \Leftrightarrow \alpha[a_1-\gamma_1]+(1-\alpha)[b_1+\beta_1] \sim \alpha[a_2-\gamma_2]+(1-\alpha)[b_2+\beta_2] \Leftrightarrow \mathfrak{R}(C) \sim \mathfrak{R}(D) \Leftrightarrow C \sim D$.

6. Examples and Results

In this section, the correct ordering of fuzzy numbers, are obtained. Also, in the Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [23].

Example 7. Let $A = (0.1, 0.3, 0.3, 0.5; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = min\{1, 1\} = 1$

$*$ step 2.

$$\mathfrak{R}(A) = \alpha[0.1-0.3]+(1-\alpha)[0.3+0.5] = 0.8-\alpha\ ,$$

and

$$\mathfrak{R}(B) = \alpha 0.2-0.3]+(1-\alpha)[0.3+0.4] = 0.7-0.8\alpha\ .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\mathfrak{R}(A) = 0.8\ ,$$

and

$$\mathfrak{R}(B) = 0.7\ .$$

Then $\mathfrak{R}(A) > \mathfrak{R}(B) \Rightarrow A > B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\mathfrak{R}(A) = -0.2\ ,$$

and

$$\mathfrak{R}(B) = -0.1\ .$$

Then $\mathfrak{R}(A) < \mathfrak{R}(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\mathfrak{R}(A) = 0.3\ ,$$

and

$$\mathfrak{R}(B) = 0.3\ .$$

Then $\mathfrak{R}(A) = \mathfrak{R}(B) \Rightarrow A \sim B$.

Example 8. Let $A = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = min\{0.8, 1\} = 0.8$

$*$ step 2.

$$\Re(A) = 0.8\alpha[0.1 - 0.3] + 0.8(1 - \alpha)[0.3 + 0.5] = 0.64 - 0.8\alpha \ ,$$

and

$$\Re(B) = 0.8\alpha[0.1 - 0.3] + 0.8(1 - \alpha)[0.3 + 0.5] = 0.64 - 0.8\alpha \ .$$

Then $\Re(A) = \Re(B) \Rightarrow \forall\alpha \ A \sim B$.

Example 9. Let $A = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $B = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = min\{0.35, 0.7\} = 0.35$

$*$ step 2.

$$\Re(A) = 0.35\alpha[-0.8 + 0.4] + 0.35(1 - \alpha)[-0.6 - 0.2] = 0.14\alpha - 0.28 \ ,$$

and

$$\Re(B) = 0.35\alpha[-0.4 + 0.2] + 0.35(1 - \alpha)[-0.3 - 0.1] = 0.07\alpha - 0.14 \ .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\Re(A) = -0.28 \ ,$$

and

$$\Re(B) = -0.14 \ .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\Re(A) = -0.14 \ ,$$

and

$$\Re(B) = -0.07 \ .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\Re(A) = -0.21 \ ,$$

and

$$\Re(B) = -0.105 \ .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

Example 10. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = min\{0.35, 0.7\} = 0.35$

$*$ step 2.

$$\Re(A) = 0.35\alpha[0.2 - 0.6] + 0.35(1-\alpha)[0.4 + 0.8] = 0.42 - 0.56\alpha\ ,$$

and

$$\Re(B) = 0.35\alpha[0.1 - 0.3] + 0.35(1-\alpha)[0.2 + 0.4] = 0.21 - 0.28\alpha\ .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\Re(A) = 0.42\ ,$$

and

$$\Re(B) = 0.21\ .$$

Then $\Re(A) > \Re(B) \Rightarrow A > B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\Re(A) = -0.14\ ,$$

and

$$\Re(B) = -0.07\ .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\Re(A) = 0.14\ ,$$

and

$$\Re(B) = 0.07\ .$$

Then $\Re(A) > \Re(B) \Rightarrow A > B$.

Example 11. Let $A = (1, 1, 1, 1; w_1)$ and $B = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = w$

$*$ step 2.

$$\Re(A) = \alpha w[1 - 1] + (1-\alpha)w[1 + 1] = 2w(1-\alpha)\ ,$$

and

$$\Re(B) = \alpha w[1 - 1] + (1-\alpha)w[1 + 1] = 2w(1-\alpha)\ .$$

Then $\Re(A) = \Re(B) \Rightarrow \forall\alpha\ A \sim B$.

Example 12. Let $A = (-0.4, -0.2, -0.1, 0.7; w_1)$ and $B = (-0.4, -0.2, -0.1, 0.7; w_2)$ be two generalized trapezoidal fuzzy numbers. Since

$*$ step 1. $w = min\{w_1, w_2\} = w$

$*$ step 2.

$$\Re(A) = \alpha w[-0.4 + 0.1] + (1 - \alpha)w[-0.2 + 0.7] = 0.5 - 0.8\alpha w\ ,$$

and

$$\Re(B) = \alpha w[-0.4 + 0.1] + (1 - \alpha)w[-0.2 + 0.7] = 0.5 - 0.8\alpha w\ .$$

Then $\Re(A) = \Re(B) \Rightarrow \forall\alpha\ A \sim B$.

6.1. **Testimony of Proposed Ranking Function.** Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [25]

Table (1): Fulfilment of the axioms for the ordering in the first and second class [25]

Index	A_1	A_2	A_3	A_4	A'_4	A_5	A_6	A'_6	A_7
Y_1	Y	Y	Y	Y	Y	Y	N	N	N
Y_2	Y	Y	Y	Y	Y	Y	N	N	N
Y_3	Y	Y	Y	N	N	Y	N	N	N
Y_4	Y	Y	Y	Y	Y	Y	N	N	N
C	Y	Y	Y	N	N	Y	N	N	N
FR	Y	Y	Y	Y	Y	Y	Y	Y	N
CL	Y	Y	Y	Y	Y	Y	Y	Y	N
LW^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_1^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_2^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
K	Y	Y	Y	N	N	N	N	N	N
W	Y	Y	Y	Y	N	N	N	N	N
J^k	Y	Y	Y	Y	Y	N	N	N	N
CH^k	Y	Y	Y	Y	Y	N	N	N	N
KP^k	Y	Y	Y	Y	Y	N	N	N	N
α	Y	Y	Y	Y	Y	Y	Y	Y	N
Proposed Approach	Y	Y	Y	Y	Y	Y	Y	Y	Y

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