Soft semi separation axioms and some types of soft functions


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Abstract. Shabir and Naz in [25] introduced the notion of soft topological spaces. They defined basic notions of soft topological spaces such as open soft sets, closed soft sets, soft subspaces, soft closure, soft nbd of a soft point, soft separation axioms, soft regular spaces, soft normal spaces and they established their several properties. Min in [19] investigated some properties of such soft separation axioms. In the present paper, we have continued to study the properties of soft topological spaces. We introduce new soft separation axioms based on the semi open soft sets which are more general than of the open soft sets. We show that the properties of soft semi $T_i$-spaces ($i = 1, 2$) are soft topological properties under the bijection and irresolute open soft mapping. Also, the property of being soft semi regular and soft semi normal are soft topological properties under bijection, irresolute soft and irresolute open soft functions. Further, we show that the properties of being soft semi $T_i$-spaces ($i = 1, 2, 3, 4$) are hereditary properties.

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [21] in 1999 as a general mathematical tool for dealing with uncertain objects. In [21, 20], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration,
probability, theory of measurement, and so on. After presentation of the operations of soft sets [17], the properties and applications of soft set theory have been studied increasingly [4, 13, 20, 23]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 3, 5, 8, 15, 16, 17, 18, 20, 22, 24, 29]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].

Recently, in 2011, Shabir and Naz [25] initiated the study of soft topological spaces. They defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbld of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [19] investigate some properties of these soft separation axioms mentioned in [25]. Banu and Halis in [7] studied some properties of soft Hausdorff space.

The main purpose of this paper is to introduce the notion of soft semi separation axioms. In particular we study the properties of the soft semi regular spaces and soft semi normal spaces. We show that if \( x_E \) is semi closed soft set for all \( x \in X \) in a soft topological space \((X, \tau, E)\), then \((X, \tau, E)\) is soft semi \( T_1 \)-space. Also, we show that if a soft topological space \((X, \tau, E)\) is soft semi \( T_3 \)-space, then \( \forall x \in X, x_E \) is semi closed soft set. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

**Definition 2.1.** [21] Let \( X \) be an initial universe and \( E \) be a set of parameters. Let \( P(X) \) denote the power set of \( X \) and \( A \) be a non-empty subset of \( E \). A pair \((F, A)\) denoted by \( F_A \) is called a soft set over \( X \), where \( F \) is a mapping given by \( F : A \to P(X) \). In other words, a soft set over \( X \) is a parametrized family of subsets of the universe \( X \). For a particular \( e \in A \), \( F(e) \) may be considered the set of \( e \)-approximate elements of the soft set \((F, A)\). The set of all these soft sets over \( X \) denoted by \( SS(X)_A \).

**Definition 2.2.** [17] Let \( F_A, G_B \in SS(X)_E \). Then \( F_A \) is soft subset of \( G_B \), denoted by \( F_A \subseteq G_B \), if

1. \( A \subseteq B \), and
2. \( F(e) \subseteq G(e), \forall e \in A \).

In this case, \( F_A \) is said to be a soft subset of \( G_B \) and \( G_B \) is said to be a soft superset of \( F_A \).

**Definition 2.3.** [17] Two soft subset \( F_A \) and \( G_B \) over a common universe set \( X \) are said to be soft equal if \( F_A \) is soft subset of \( G_B \) and \( G_B \) is soft subset of \( F_A \).

**Definition 2.4.** [4] The complement of a soft set \((F, A)\), denoted by \((F, A)^c\), is defined by \((F, A)^c = (F^c, A)\), \( F^c : A \to P(X) \) is a mapping given by \( F^c(e) = X - F(e), \forall e \in A \) and \( F^c \) is called the soft complement function of \( F \).

Clearly \((F^c)^c\) is the same as \( F \) and \(((F, A)^c)^c = (F, A)\).
Definition 2.5. [25] The difference of two soft sets \((F, E)\) and \((G, E)\) over the common universe \(X\), denoted by \((F, E) - (G, E)\) is the soft set \((H, E)\) where for all \(e \in E\), \(H(e) = F(e) - G(e)\).

Definition 2.6. [25] Let \((F, E)\) be a soft set over \(X\) and \(x \in X\). We say that \(x \in (F, E)\) read as \(x\) belongs to the soft set \((F, E)\) whenever \(x \in F(e)\) for all \(e \in E\).

Definition 2.7. [25] The soft set \((F, E)\) over \(X\) such that \(F(e) = \{x\}\), \(\forall e \in E\), is called singleton soft point and denoted by \(x_E\) or \((x, E)\).

Definition 2.8. [17] A soft set \((F, A)\) over \(X\) is said to be a NULL soft set denoted by \(\tilde{\phi}\) or \(\phi_A\) if for all \(e \in A\), \(F(e) = \phi\).

Definition 2.9. [17] A soft set \((F, A)\) over \(X\) is said to be an absolute soft set denoted by \(\bar{A}\) or \(X_A\) if for all \(e \in A\), \(F(e) = X\). Clearly, we have \(X_{\bar{A}} = \phi\) and \(\phi_{\bar{A}} = X_A\).

Definition 2.10. [17] The union of two soft sets \((F, A)\) and \((G, B)\) over the common universe \(X\) is the soft set \((H, C)\), where \(C = A \cup B\) and for all \(e \in C\),

\[
H(e) = \begin{cases} 
F(e), & e \in A - B, \\
G(e), & e \in B - A, \\
F(e) \cup G(e), & e \in A \cap B.
\end{cases}
\]

Definition 2.11. [17] The intersection of two soft sets \((F, A)\) and \((G, B)\) over the common universe \(X\) is the soft set \((H, C)\), where \(C = A \cap B\) and for all \(e \in C\), \(H(e) = F(e) \cap G(e)\).

Note that, in order to efficiently discuss, we consider only soft sets \((F, E)\) over a universe \(X\) in which all the parameter sets \(E\) are same. We denote the family of these soft sets by \(SS(X)_E\).

Definition 2.12. [30] Let \(I\) be an arbitrary indexed set and \(L = \{(F, E)_i, i \in I\}\) be a subfamily of \(SS(X)_E\).

\[
1):\text{ The union of } L \text{ is the soft set } (H, E), \text{ where } H(e) = \bigcup_{i \in I} F_i(e) \text{ for each } e \in E. \text{ We write } \bigcup_{i \in I} (F, E)_i = (H, E).
\]

\[
2):\text{ The intersection of } L \text{ is the soft set } (M, E), \text{ where } M(e) = \bigcap_{i \in I} F_i(e) \text{ for each } e \in E. \text{ We write } \bigcap_{i \in I} (F, E)_i = (M, E).
\]

Definition 2.13. [25] Let \(\tau\) be a collection of soft sets over a universe \(X\) with a fixed set of parameters \(E\), then \(\tau \subseteq SS(X)_E\) is called a soft topology on \(X\) if

\[
1):\text{ } \tilde{X}, \tilde{\phi} \in \tau, \text{ where } \tilde{\phi}(e) = \phi \text{ and } \tilde{X}(e) = X, \forall e \in E,
\]

\[
2):\text{ the union of any number of soft sets in } \tau \text{ belongs to } \tau,
\]

\[
3):\text{ the intersection of any two soft sets in } \tau \text{ belongs to } \tau.
\]

The triplet \((X, \tau, E)\) is called a soft topological space over \(X\).

Definition 2.14. [10] Let \((X, \tau, E)\) be a soft topological space. A soft set \((F, A)\) over \(X\) is said to be closed soft set in \(X\), if its relative complement \((F, A)^c\) is open soft set.
Definition 2.15. [10] Let \((X, \tau, E)\) be a soft topological space. The members of \(\tau\) are said to be open soft sets in \(X\). We denote the set of all open soft sets over \(X\) by \(OS(X, \tau, E)\), or \(OS(X)\) and the set of all closed soft sets by \(CS(X, \tau, E)\), or \(CS(X)\).

Definition 2.16. [25] Let \((X, \tau, E)\) be a soft topological space and \((F, E) \in SS(X)_E\). The soft closure of \((F, E)\), denoted by \(\text{cl}(F, E)\) is the intersection of all closed soft super sets of \((F, E)\), i.e.,

\[
\text{cl}(F, E) = \bigcap \{ (H, E) : (H, E) \text{ is closed soft set and } (F, E) \subseteq (H, E) \}.
\]

Definition 2.17. [30] Let \((X, \tau, E)\) be a soft topological space and \((F, E) \in SS(X)_E\). The soft interior of \((G, E)\), denoted by \(\text{int}(G, E)\) is the union of all open soft subsets of \((G, E)\), i.e.,

\[
\text{int}(G, E) = \bigcup \{ (H, E) : (H, E) \text{ is open soft set and } (H, E) \subseteq (G, E) \}.
\]

Definition 2.18. [30] The soft set \((F, E) \in SS(X)_E\) is called a soft point in \(X_E\) if there exist \(x \in X\) and \(e \in E\) such that \(F(e) = \{x\}\) and \(F(e') = \phi\) for each \(e' \in E - \{e\}\), and the soft point \((F, E)\) is denoted by \(x_e\).

Definition 2.19. [30] The soft point \(x_e\) is said to be belonging to the soft set \((G, A)\), denoted by \(x_e \in (G, A)\), if for the element \(e \in A\), \(F(e) \subseteq G(e)\).

Proposition 2.1. [26] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.

Definition 2.20. [25] Let \((X, \tau, E)\) be a soft topological space, \((F, E) \in SS(X)_E\) and \(Y\) be a non null subset of \(X\). Then the sub soft set of \((F, E)\) over \(Y\) denoted by \((F_Y, E)\), is defined as follows:

\[
(F_Y, E) = Y \cap F(e) \quad \forall e \in E.
\]

In other words \((F_Y, E) = \hat{Y} \cap (F, E)\).

Definition 2.21. [25] Let \((X, \tau, E)\) be a soft topological space and \(Y\) be a non null subset of \(X\). Then \(\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}\) is called the soft relative topology on \(Y\) and \((Y, \tau_Y, E)\) is called a soft subspace of \((X, \tau, E)\).

Theorem 2.1. [25] Let \((Y, \tau_Y, E)\) be a soft subspace of a soft topological space \((X, \tau, E)\) and \((F, E) \in SS(X)_E\). Then

1. If \((F, E)\) is open soft set in \(Y\) and \(\hat{Y} \in \tau\), then \((F, E) \in \tau\).
2. If \((F, E)\) is open soft set in \(Y\) if and only if \((F, E) = \hat{Y} \cap (G, E)\) for some \((G, E) \in \tau\).
3. If \((F, E)\) is closed soft set in \(Y\) if and only if \((F, E) = \hat{Y} \cap (H, E)\) for some \((H, E)\) is \(\tau\)-closed soft set.

Definition 2.22. [11] Let \((X, \tau, E)\) be a soft topological space and \((F, E) \in SS(X)_E\). If \((F, E) \subseteq \text{cl}(int(F, E))\), then \((F, E)\) is called semi-open soft set. We denote the set of all semi-open soft sets by \(\text{SOS}(X, \tau, E)\), or \(\text{SOS}(X)\) and the set of all semi-closed soft sets by \(\text{SCS}(X, \tau, E)\), or \(\text{SCS}(X)\).
Definition 2.23. [2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets on $X$ and $Y$ respectively, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

1. If $(F, A) \in SS(X)_A$. Then the image of $(F, A)$ under $f_{pu}$, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is soft set in $SS(Y)_B$ such that $f_{pu}(F)(b) = \{ \cup_{a \in p^{-1}(b) \cap A} u(F(a)), \quad p^{-1}(b) \cap A \neq \phi, \quad \text{otherwise.} \}$ for all $b \in B$.

2. If $(G, B) \in SS(Y)_B$. Then the inverse image of $(G, B)$ under $f_{pu}$, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is soft set in $SS(X)_A$ such that $f_{pu}^{-1}(G)(a) = \{ u^{-1}(G(p(a))), \quad p(a) \in B, \quad \phi, \quad \text{otherwise.} \}$ for all $a \in A$.

The soft function $f_{pu}$ is called surjective if $p$ and $u$ are surjective, also it is said to be injective if $p$ and $u$ are injective.

Definition 2.24. [11, 14, 30] Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, The function $f_{pu}$ is said to be

1. Continuous soft if $f_{pu}^{-1}(G, B) \in \tau_1 \forall (G, B) \in \tau_2$.
2. Open soft if $f_{pu}(G, A) \in \tau_2 \forall (G, A) \in \tau_1$.
3. Semi open soft if $f_{pu}(G, A) \in SOS(Y) \forall (G, A) \in \tau_1$.
4. Semi continuous soft function if $f_{pu}^{-1}(G, B) \in SOS(X) \forall (G, B) \in \tau_2$.
5. Irresolute soft if $f_{pu}^{-1}(G, B) \in SOS(X) \forall (G, B) \in SOS(Y) [f_{pu}(F, B) \in SCS(X) \forall (F, B) \in SCS(Y)]$.
6. Irresolute open soft (resp. irresolute closed soft) if $f_{pu}(G, A) \in SOS(Y) \forall (G, A) \in SOS(X)$ (resp. $f_{pu}(F, A) \in SCS(Y) \forall (F, A) \in SCS(Y)$).

Theorem 2.2. [2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements hold,

(a): $f_{pu}^{-1}((G, B) \cup (G, B)) = (f_{pu}^{-1}(G, B)) \cup (G, B) \in SS(Y)_B$.

(b): $f_{pu}(f_{pu}^{-1}((G, B))) \subseteq (G, B) \forall (G, B) \in SS(Y)_B$. If $f_{pu}$ is surjective, then the equality holds.

(c): $(F, A) \subseteq f_{pu}^{-1}(f_{pu}(F, A)) \forall (F, A) \in SS(X)_A$. If $f_{pu}$ is injective, then the equality holds.

(d): $f_{pu}(\tilde{X}) \subseteq \tilde{Y}$. If $f_{pu}$ is surjective, then the equality holds.

(e): $f_{pu}(\hat{Y}) = \hat{X}$ and $f_{pu}(\hat{F}_A) = \hat{F}_B$.

(f): If $(F, A) \subseteq (G, A)$, then $f_{pu}(F, A) \subseteq f_{pu}(G, A)$.

(g): If $(F, B) \subseteq (G, B)$, then $f_{pu}^{-1}(F, B) \subseteq f_{pu}^{-1}(G, B) \forall (F, B), (G, B) \in SS(Y)_B$.

(h): $f_{pu}^{-1}[(F, B) \cap (G, B)] = f_{pu}^{-1}(F, B) \cap f_{pu}^{-1}(G, B)$ and $f_{pu}^{-1}[(F, B) \cap (G, B)] = f_{pu}^{-1}(F, B) \cap f_{pu}^{-1}(G, B) \forall (F, B), (G, B) \in SS(Y)_B$.

(i): $f_{pu}(F, A) \cup (G, A) = f_{pu}(F, A) \cup f_{pu}(G, A)$ and $f_{pu}[(F, A) \cap (G, A)] \subseteq f_{pu}(F, A) \cap f_{pu}(G, A) \forall (F, A), (G, A) \in SS(X)_A$. If $f_{pu}$ is injective, then the equality holds.
3. Soft semi separation axioms

**Definition 3.1.** Let \((X, \tau, E)\) be a soft topological space and \(x, y \in X\) such that \(x \neq y\). Then \((X, \tau, E)\) is called a soft semi \(T_o\)-space if there exist semi open soft sets \((F, E)\) and \((G, E)\) such that either \(x \in (F, E)\) and \(y \notin (F, E)\) or \(y \in (G, E)\) and \(x \notin (G, E)\).

**Proposition 3.1.** Let \((X, \tau, E)\) be a soft topological space and \(x, y \in X\) such that \(x \neq y\). If there exist semi open soft sets \((F, E)\) and \((G, E)\) such that either \(x \in (F, E)\) and \(y \notin (F, E)\) or \(y \in (G, E)\) and \(x \notin (G, E)\). Then \((X, \tau, E)\) is soft semi \(T_o\)-space.

**Proof.** Let \(x, y \in X\) such that \(x \neq y\). Let \((F, E)\) and \((G, E)\) be semi open soft sets such that either \(x \in (F, E)\) and \(y \in (F, E)^c\) or \(y \in (G, E)\) and \(x \in (G, E)^c\). If \(x \in (F, E)\) and \(y \in (F, E)^c\). Then \(y \notin (F(e))^c\) for all \(e \in E\). This implies that, \(y \notin F(e)\) for all \(e \in E\). Therefore, \(y \notin (F, E)\). Similarly, if \(y \in (G, E)\) and \(x \notin (G, E)^c\), then \(x \notin (G, E)\). Hence \((X, \tau, E)\) is soft semi \(T_o\)-space.

**Theorem 3.1.** A soft subspace \((Y, \tau_Y, E)\) of a soft semi \(T_o\)-space \((X, \tau, E)\) is soft semi \(T_o\).

**Proof.** Let \(x, y \in Y\) such that \(x \neq y\). Then \(x, y \in X\) such that \(x \neq y\). Hence there exist semi open soft sets \((F, E)\) and \((G, E)\) in \(X\) such that either \(x \in (F, E)\) and \(y \notin (F, E)\) or \(y \in (G, E)\) and \(x \notin (G, E)\). Since \(x \in Y\). Then \(x \in \hat{Y}\). Hence \(x \in \hat{Y} \cap (F, E) = (F_Y, E)\). \((F, E)\) is semi open soft set. Consider \(y \notin (F, E)\). This implies that, \(y \notin F(e)\) for some \(e \in E\). Therefore, \(y \notin \hat{Y} \cap (F, E) = (F_Y, E)\). Similarly, if \(y \in (G, E)\) and \(x \notin (G, E)^c\), then \(y \notin (G_Y, E)\) and \(x \notin (G_Y, E)^c\). Thus, \((Y, \tau_Y, E)\) is soft semi \(T_o\).

**Definition 3.2.** Let \((X, \tau, E)\) be a soft topological space and \(x, y \in X\) such that \(x \neq y\). Then \((X, \tau, E)\) is called a soft semi \(T_1\)-space if there exist semi open soft sets \((F, E)\) and \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (F, E)\) and \(y \in (G, E)\) and \(x \notin (G, E)\).

**Proposition 3.2.** Let \((X, \tau, E)\) be a soft topological space and \(x, y \in X\) such that \(x \neq y\). If there exist semi open soft sets \((F, E)\) and \((G, E)\) such that \(x \in (F, E)\) and \(y \in (F, E)^c\) and \(y \in (G, E)\) and \(x \in (G, E)^c\). Then \((X, \tau, E)\) is soft semi \(T_1\)-space.

**Proof.** It is similar to the proof of Theorem 3.1.

**Theorem 3.2.** A soft subspace \((Y, \tau_Y, E)\) of a soft semi \(T_1\)-space \((X, \tau, E)\) is soft semi \(T_1\).

**Proof.** It is similar to the proof of Theorem 3.1.

**Theorem 3.3.** Let \((X, \tau, E)\) be a soft topological space. If \(x_E\) is semi closed soft set in \(\tau\) for all \(x \in X\), then \((X, \tau, E)\) is soft semi \(T_1\)-space.

**Proof.** Suppose that \(x \in X\) and \(x_E\) is semi closed soft set in \(\tau\). Then \(x_E^c\) is semi open soft set in \(\tau\). Let \(x, y \in X\) such that \(x \neq y\). For \(x \in X\) and \(x_E^c\) is semi open soft set such that \(x \notin x_E^c\) and \(y \in x_E^c\). Similarly, \(y_E^c\) is semi open soft set in \(\tau\) such that \(y \notin y_E^c\) and \(x \in y_E^c\). Thus, \((X, \tau, E)\) is soft semi \(T_1\)-space over \(X\).
Definition 3.3. Let \((X, \tau, E)\) be a soft topological space and \(x, y \in X\) such that \(x \neq y\). Then \((X, \tau, E)\) is called a soft semi Hausdorff space or a soft semi \(T_2\)-space if there exist semi open soft sets \((F, E)\) and \((G, E)\) such that \(x \in (F, E), y \in (G, E)\) and \((F, E) \cap (G, E) = \emptyset\).

Theorem 3.4. For a soft topological space \((X, \tau, E)\) we have:
soft semi \(T_2\)-space \(\Rightarrow\) soft semi \(T_1\)-space \(\Rightarrow\) soft semi \(T_0\)-space.

Proof. Straightforward.

Remark 3.1. The converse of Theorem 3.4 is not true in general, as shown in the following examples.

Examples 3.1. (1): Let \(X = \{h_1, h_2\}, E = \{e_1, e_2\}\) and \(\tau = \{\hat{X}, \hat{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}\) where \((F_1, E), (F_2, E), (F_3, E)\) are soft sets over \(X\) defined as follows:
\[F_1(e_1) = X, \quad F_1(e_2) = \{h_2\},\]
\[F_2(e_1) = \{h_1\}, \quad F_2(e_2) = X,\]
\[F_3(e_1) = \{h_1\}, \quad F_3(e_2) = \{h_2\}.\]
Then \(\tau\) defines a soft topology on \(X\). Also \((X, \tau, E)\) is soft semi \(T_1\)-space but it is not a soft semi \(T_2\)-space, for \(h_1, h_2 \in X\) and \(h_1 \neq h_2\), but there is no semi open soft sets \((F, E)\) and \((G, E)\) such that \(h_1 \in (F, E), h_2 \in (G, E)\) and \((F, E) \cap (G, E) = \emptyset\).

(2): Let \(X = \{h_1, h_2\}, E = \{e_1, e_2\}\) and \(\tau = \{\hat{X}, \hat{\emptyset}, (F_1, E)\}\) where \((F_1, E)\) is soft set over \(X\) defined as follows by \(F_1(e_1) = X, \quad F_1(e_2) = \{h_2\}\). Then \(\tau\) defines a soft topology on \(X\). Also \((X, \tau, E)\) is soft semi \(T_0\)-space but not a soft semi \(T_1\)-space, since \(h_1, h_2 \in X, h_1 \neq h_2\), but all the open soft sets which contain \(h_1\) also contain \(h_2\).

Theorem 3.5. A soft subspace \((Y, \tau_Y, E)\) of a soft semi \(T_2\)-space \((X, \tau, E)\) is soft semi \(T_2\).

Proof. Let \(x, y \in Y\) such that \(x \neq y\). Then \(x, y \in X\) such that \(x \neq y\). Hence there exist semi open soft sets \((F, E)\) and \((G, E)\) in \(X\) such that \(x \in (F, E), y \in (G, E)\) and \((F, E) \cap (G, E) = \emptyset\). It follows that \(x \in F(e), y \in G(e)\) and \(F(e) \cap G(e) = \emptyset\) for all \(e \in E\). This implies that, \(x \in Y \cap F(e), y \in Y \cap G(e)\) and \(F(e) \cap G(e) = \emptyset\). Thus, \(x \in \hat{Y} \cap (F, E) = (F_Y, E), y \in \hat{Y} \cap (G, E) = (G_Y, E)\) and \((F_Y, E) \cap (G_Y, E) = \emptyset\), where \((F_Y, E), (G_Y, E)\) are semi open soft sets in \(Y\). Therefore, \((Y, \tau_Y, E)\) is soft semi \(T_2\)-space.

Definition 3.4. Let \((X, \tau, E)\) be a soft topological space, \((G, E)\) be a semi closed soft set in \(X\) and \(x \in X\) such that \(x \notin (G, E)\). If there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \(x \in (F_1, E), (G, E) \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \emptyset\), then \((X, \tau, E)\) is called a soft semi regular space. A soft semi regular \(T_1\)-space is called a soft semi \(T_3\)-space.

Proposition 3.3. Let \((X, \tau, E)\) be a soft topological space, \((G, E)\) be a semi closed soft set in \(X\) and \(x \in X\) such that \(x \notin (G, E)\). If \((X, \tau, E)\) is soft semi regular space, then there exists a semi open soft set \((F, E)\) such that \(x \in (F, E)\) and \((F, E) \cap (G, E) = \emptyset\).

Proof. It is obvious from Definition 3.4.
Proposition 3.4. Let \((X, \tau, E)\) be a soft topological space, \((F, E) \in SS(X)_E\) and \(x \in X\). Then:

1. \(x \in (F, E)\) if and only if \(x \in \mathcal{F}(F, E)\).
2. If \(x \in \mathcal{F}(F, E) = \emptyset\), then \(x \notin (F, E)\).

Proof. Obvious.

Theorem 3.6. Let \((X, \tau, E)\) be a soft topological space and \(x \in X\). If \((X, \tau, E)\) is soft semi regular space, then:

1. \(x \notin (F, E)\) if and only if \(x \in \mathcal{F}(F, E)\) for every semi closed soft set \((F, E)\).
2. \(x \notin (G, E)\) if and only if \(x \in \mathcal{F}(G, E)\) for every semi open soft set \((G, E)\).

Proof.

(1): Let \((F, E)\) be a semi closed soft set such that \(x \notin (F, E)\). Since \((X, \tau, E)\) is soft semi regular space. Then by Proposition 3.3 there exists a semi open soft set \((G, E)\) such that \(x \in (G, E)\) and \((F, E)\) \(\cap\) \((G, E) = \emptyset\). It follows that \(x \in \mathcal{F}(G, E)\) from Proposition 3.4 (1). Hence \(x \notin \mathcal{F}(F, E) = \emptyset\). Conversely, if \(x \in \mathcal{F}(F, E) = \emptyset\), then \(x \notin (F, E)\) from Proposition 3.4 (2).

(2): Let \((G, E)\) be a semi open soft set such that \(x \notin (G, E)\). If \(x \notin G(e)\) for all \(e \in E\), then we get the proof. If \(x \notin G(e_1)\) and \(x \in G(e_2)\) for some \(e_1, e_2 \in E\), then \(x \in G^c(e_1)\) and \(x \notin G^c(e_2)\) for some \(e_1, e_2 \in E\). This means that, \(x \in \mathcal{F}(G, E)\). Hence \((G, E)^c\) is semi closed soft set such that \(x \notin (G, E)^c\). It follows by (1) \(x \in \mathcal{F}(G, E)^c = \emptyset\). This implies that, \(x \in \mathcal{F}(G, E)\) and so \(x \in (G, E)\), which is contradiction with \(x \notin (G, E)\). Therefore, \(x \in \mathcal{F}(G, E) = \emptyset\). Conversely, if \(x \in \mathcal{F}(G, E) = \emptyset\), then it is obvious that \(x \notin (G, E)\). This completes the proof.

Corollary 3.1. Let \((X, \tau, E)\) be a soft topological space and \(x \in X\). If \((X, \tau, E)\) is soft semi regular space, then the following are equivalent:

1. \((X, \tau, E)\) is soft semi \(T_1\) space.
2. \(\forall x, y \in X\) such that \(x \neq y\), there exist semi open soft sets \((F, E)\) and \((G, E)\) such that \(x \in \mathcal{F}(F, E)\) and \(y \in \mathcal{F}(G, E)\) and \(x \notin \mathcal{F}(G, E)\) and \(y \in \mathcal{F}(F, E)\).

Proof. It is obvious from Theorem 3.6.

Theorem 3.7. Let \((X, \tau, E)\) be a soft topological space and \(x \in X\). Then the following are equivalent:

1. \((X, \tau, E)\) is soft semi regular space.
2. For every semi closed soft set \((G, E)\) such that \(x \in \mathcal{F}(G, E) = \emptyset\), there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \(x \in \mathcal{F}(F_1, E)\), \((G, E)\) \(\subseteq\) \((F_2, E)\) and \((F_1, E)\) \(\cap\) \((F_2, E) = \emptyset\).

Proof.

(1) \(\Rightarrow\) (2): Let \((G, E)\) be a semi closed soft set such that \(x \in \mathcal{F}(G, E) = \emptyset\). Then \(x \notin (G, E)\) from Theorem 3.6 (1). It follows by (1), there exist semi
open soft sets \((F_1, E)\) and \((F_2, E)\) such that \(x \in (F_1, E)\), \((G, E) \supseteq (F_2, E)\) and \((F_1, E)\cap(F_2, E) = \phi\). This means that, \(x_E \subseteq (F_1, E)\), \((G, E) \subseteq (F_2, E)\) and \((F_1, E)\cap(F_2, E) = \phi\).

\((2) \Rightarrow (1)\) : Let \((G, E)\) be a semi closed soft set such that \(x \notin (G, E)\). Then \(x_E \cap (G, E) = \phi\) from Theorem 3.6 (1). It follows by (2), there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \(x_E \subseteq (F_1, E)\), \((G, E) \subseteq (F_2, E)\) and \((F_1, E)\cap(F_2, E) = \phi\). Hence \(x \in (F_1, E)\), \((G, E) \subseteq (F_2, E)\) and \((F_1, E)\cap(F_2, E) = \phi\). Thus, \((X, \tau, E)\) is soft semi regular space.

**Theorem 3.8.** Let \((X, \tau, E)\) be a soft topological space. If \((X, \tau, E)\) is soft semi \(T_3\)-space, then \(\forall x \in X\), \(x_E = \text{semi closed soft set}\).

**Proof.** We want to prove that \(x_E\) is semi closed soft set, which is sufficient to prove that \(x_E\) is semi open soft set for all \(y \in \{x\}^c\). Since \((X, \tau, E)\) is soft semi \(T_3\)-space, then there exist semi open soft sets \((F, E)\) and \((G, E)\) such that \(y_E \subseteq (F, E)\) and \(x_E \cap (F, E) = \phi\). Now we want to prove that \(x_E \subseteq \bigcup_{y \in \{x\}^c} (F, E)\). Let \(\bigcup_{y \in \{x\}^c} (F, E) = (H, E)\). Since \(x_E \subseteq (F, E)\), \(H(e) = \bigcup_{y \in \{x\}^c} (F, E)\). Let \(x_E \subseteq \bigcup_{y \in \{x\}^c} (F, E)\). Then \(x \in (F_1, E)\), \(y \in (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Thus, \((X, \tau, E)\) is soft semi \(T_2\)-space.

**Theorem 3.9.** Every soft semi \(T_3\)-space is soft semi \(T_2\)-space.

**Proof.** Let \((X, \tau, E)\) be a soft semi \(T_3\)-space and \(x, y \in X\) such that \(x \neq y\). By Theorem 3.8, \(y_E\) is semi closed soft set and \(x \notin y_E\). It follows from the soft semi regularity, there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \(x \in (F_1, E)\), \(y_E \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Thus, \(x \in (F_1, E)\), \(y \in (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Therefore, \((X, \tau, E)\) is soft semi \(T_2\)-space.

**Theorem 3.10.** A soft subspace \((Y, \tau_Y, E)\) of a soft semi \(T_3\)-space \((X, \tau, E)\) is soft semi \(T_3\).

**Proof.** By Theorem 3.1 \((Y, \tau_Y, E)\) is soft semi \(T_1\)-space. Now we want to prove that \((Y, \tau_Y, E)\) is soft semi regular space. Let \(y \in Y\) and \((G, E)\) be a semi closed soft set in \(Y\) such that \(y \notin (G, E)\). Then \((G, E) = (Y, E) \cap (F, E)\) for some semi closed soft set \((F, E)\) in \(X\) from Theorem 2.1. Hence \(y \notin (Y, E) \cap (F, E)\). But \(y \in (Y, E)\), so \(y \notin (F, E)\). Since \((X, \tau, E)\) is soft semi \(T_3\)-space, so there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) in \(X\) such that \(y \in (F_1, E)\), \((F, E) \supseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Take \((G_1, E) = (Y, E) \cap (F_1, E)\) and \((G_2, E) = (Y, E) \cap (F_2, E)\) then \((G_1, E), (G_2, E)\) are semi open soft sets in \(Y\) such that \(y \in (G_1, E)\), \((G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)\) and \((G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi\). Thus, \((Y, \tau_Y, E)\) is soft semi \(T_3\)-space.

**Definition 3.5.** Let \((X, \tau, E)\) be a soft topological space, \((F, E)\), \((G, E)\) be semi closed soft sets in \(X\) such that \((F, E) \cap (G, E) = \phi\). If there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \((F, E) \subseteq (F_1, E)\), \((G, E) \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\), then \((X, \tau, E)\) is called a soft semi normal space. A soft semi normal \(T_1\)-space is called a soft semi \(T_4\)-space.
Theorem 3.11. Let \((X, \tau, E)\) be a soft topological space and \(x \in X\). Then the following are equivalent:

1. \((X, \tau, E)\) is soft semi normal space.
2. For every semi closed soft set \((F, E)\) and semi open soft set \((G, E)\) such that \(F, E) \subseteq (G, E)\), there exists a semi open soft set \((F_1, E)\) such that \(F, E) \subseteq (F_1, E)\), \(SScl(F_1, E) \subseteq (G, E)\), where \(SScl(F_1, E)\) is the soft semi closure of \((F_1, E)\) mentioned in [11].

Proof.

(1) \Rightarrow (2): Let \((F, E)\) be a semi closed soft set and \((G, E)\) be a semi open soft set such that \(F, E) \subseteq (G, E)\). Then \((F, E), (G, E)\) are semi closed soft sets such that \(F, E) \cap (G, E) = \emptyset\). It follows by (1), there exist semi open soft sets \((F_1, E)\) and \((F_2, E)\) such that \((F, E) \subseteq (F_1, E)\), \((G, E) \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \emptyset\). Now \((F_1, E) \subseteq (F_2, E)\), so \(SScl(F_1, E) \subseteq SScl(F_2, E)\), where \((G, E)\) is semi open soft set. Also \((F_2, E) \subseteq (G, E)\). Hence \(SScl(F_1, E) \subseteq (F_2, E) \subseteq (G, E)\). Thus, \(F, E) \subseteq (F_1, E)\), \(SScl(F_1, E) \subseteq (G, E)\).

(2) \Rightarrow (1): Let \((G_1, E), (G_2, E)\) be semi closed soft sets such that \((G_1, E) \cap (G_2, E) = \emptyset\). Then \((G_1, E) \subseteq (G_2, E)\), then by hypothesis, there exists a semi open soft set \((F_1, E)\) such that \((G_1, E) \subseteq (F_1, E)\), \(SScl(F_1, E) \subseteq (G_2, E)\). So, \((G_2, E) \subseteq [SScl(F_1, E)]\), \((G_1, E) \subseteq (F_1, E)\) and \([SScl(F_1, E)] \subseteq (F_1, E)\) are semi open soft sets. Thus, \((X, \tau, E)\) is soft semi normal space.

Theorem 3.12. A semi closed soft subspace \((Y, \tau_Y, E)\) of a soft semi normal space \((X, \tau, E)\) is soft semi normal.

Proof. Let \((G_1, E), (G_2, E)\) be semi closed soft sets in \(Y\) such that \((G_1, E) \cap (G_2, E) = \emptyset\). Then \((G_1, E) = (Y, E) \cap (F_1, E)\) and \((G_2, E) = (Y, E) \cap (F_1, E)\) for some semi closed soft sets \((F_1, E)\) and \((F_2, E)\) in \(X\) from Theorem 2.1. Since \(Y\) is sa semi closed soft subset of \(X\). Then \((G_1, E), (G_2, E)\) are semi closed soft sets in \(X\) such that \((G_1, E) \cap (G_2, E) = \emptyset\). Hence by soft semi normality there exist semi open soft sets \((H_1, E)\) and \((H_2, E)\) such that \((G_1, E) \subseteq (H_1, E)\), \((G_2, E) \subseteq (H_2, E)\) and \((H_1, E) \cap (H_2, E) = \emptyset\). Since \((G_1, E), (G_2, E) \subseteq (Y, E)\), then \((G_1, E) \subseteq (Y, E)\), \((G_2, E) \subseteq (Y, E)\), \((Y, E) \cap (H_1, E) = \emptyset\), \((Y, E) \cap (H_2, E) = \emptyset\), \((Y, E) \cap (H_1, E) \cap (H_2, E) = \emptyset\). Therefore, \((Y, \tau_Y, E)\) is soft semi normal space.

Theorem 3.13. Let \((X, \tau, E)\) be a soft topological space. If \((X, \tau, E)\) is soft semi normal space and \(x_E\) is semi closed soft set in \(\tau\) for all \(x \in X\), then \((X, \tau, E)\) is soft semi \(T_3\)-space.

Proof. Since \(x_E\) is semi closed soft set for all \(x \in X\), then \((X, \tau, E)\) is soft semi \(T_1\)-space from Theorem 3.3. Also \((X, \tau, E)\) is soft semi regular space from Theorem 3.7 and Definition 3.5. Hence \((X, \tau, E)\) is soft semi \(T_3\)-space.
4. Some types of soft functions

Theorem 4.1. Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be soft topological spaces and \(f_{pu} : \text{SS}(X)_A \to \text{SS}(Y)_B\) be soft function which is bijective and irresolute open soft. If \((X, \tau_1, A)\) is soft semi \(T_o\)-space, then \((Y, \tau_2, B)\) is also a soft semi \(T_o\)-space.

Proof. Let \(y_1, y_2 \in Y\) such that \(y_1 \neq y_2\). Since \(f_{pu}\) is surjective, then \(\exists x_1, x_2 \in X\) such that \(u(x_1) = y_1, u(x_2) = y_2\) and \(x_1 \neq x_2\). By hypothesis, there exist semi open soft sets \((F, A)\) and \((G, A)\) in \(X\) such that either \(x_1 \in (F, A)\) and \(x_2 \notin (F, A)\) or \(x_2 \in (G, A)\) and \(x_1 \notin (G, A)\). So, either \(x_1 \in F_A(e)\) and \(x_2 \notin F_A(e)\) or \(x_2 \in G_A(e)\) and \(x_1 \notin G_A(e)\) for all \(e \in E\). This implies that, either \(y_1 = u(x_1) \in u[F_A(e)]\) and \(y_2 = u(x_2) \notin u[F_A(e)]\) or \(y_2 = u(x_2) \in u[G_A(e)]\) and \(y_1 = u(x_1) \notin u[G_A(e)]\) for all \(e \in E\). Hence either \(y_1 \in f_{pu}(F, A)\) and \(y_2 \notin f_{pu}(F, A)\) or \(y_2 \in f_{pu}(G, A)\) and \(y_1 \notin f_{pu}(G, A)\). Since \(f_{pu}\) is irresolute open soft function, then \(f_{pu}(F, A), f_{pu}(G, A)\) are semi open soft sets in \(Y\). Hence \((Y, \tau_2, B)\) is also a soft semi \(T_o\)-space.

Theorem 4.2. Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be soft topological spaces and \(f_{pu} : \text{SS}(X)_A \to \text{SS}(Y)_B\) be soft function which is bijective and irresolute open soft. If \((X, \tau_1, A)\) is soft semi \(T_1\)-space, then \((Y, \tau_2, B)\) is also a soft semi \(T_1\)-space.

Proof. It is similar to the proof of Theorem 4.1.

Theorem 4.3. Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be soft topological spaces and \(f_{pu} : \text{SS}(X)_A \to \text{SS}(Y)_B\) be soft function which is bijective and irresolute open soft. If \((X, \tau_1, A)\) is soft semi \(T_2\)-space, then \((Y, \tau_2, B)\) is also a soft semi \(T_2\)-space.

Proof. \(y_1, y_2 \in Y\) such that \(y_1 \neq y_2\). Since \(f_{pu}\) is surjective, then \(\exists x_1, x_2 \in X\) such that \(u(x_1) = y_1, u(x_2) = y_2\) and \(x_1 \neq x_2\). By hypothesis, there exist semi open soft sets \((F, A)\) and \((G, A)\) in \(X\) such that \(x_1 \in (F, A)\), \(x_2 \in (G, A)\) and \((F, A) \triangle (G, A) = \emptyset\). So, \(x_1 \in F_A(e)\), \(x_2 \in G_A(e)\) and \(F_A(e) \triangle G_A(e) = \emptyset\) for all \(e \in E\). This implies that, \(y_1 = u(x_1) \in u[F_A(e)]\), \(y_2 = u(x_2) \in u[G_A(e)]\) for all \(e \in E\). Hence \(y_1 \in f_{pu}(F, A)\), \(y_2 \in f_{pu}(G, A)\) and \(f_{pu}(F, A) \triangle f_{pu}(G, A) = f_{pu}[F_A(e) \triangle G_A(e)] = f_{pu}[\emptyset] = \emptyset\) from Theorem 2.2. Since \(f_{pu}\) is irresolute open soft function, then \(f_{pu}(F, A), f_{pu}(G, A)\) are semi open soft sets in \(Y\). Thus, \((Y, \tau_2, B)\) is also a soft semi \(T_2\)-space.

Theorem 4.4. Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be soft topological spaces and \(f_{pu} : \text{SS}(X)_A \to \text{SS}(Y)_B\) be soft function which is bijective, irresolute soft and irresolute open soft. If \((X, \tau_1, A)\) is soft semi regular space, then \((Y, \tau_2, B)\) is also a soft semi regular space.

Proof. Let \((G, B)\) be a semi closed soft set in \(Y\) and \(y \in Y\) such that \(y \notin (G, B)\). Since \(f_{pu}\) is surjective and irresolute soft, then \(\exists x \in X\) such that \(u(x) = y\) and \(f_{pu}^{-1}(G, B)\) is semi closed soft set in \(X\) such that \(x \notin f_{pu}^{-1}(G, B)\). By hypothesis, there exist semi open soft sets \((F, A)\) and \((H, A)\) in \(X\) such that \(x \in (F, A)\), \(f_{pu}^{-1}(G, B) \subseteq (H, A)\) and \((F, A) \triangle (H, A) = \emptyset\). It follows that \(x \in F_A(e)\) for all \(e \in E\) and \((G, B) = f_{pu}[f_{pu}^{-1}(G, B)] \subseteq f_{pu}(H, A)\) from Theorem 2.2. So, \(y = u(x_1) \in u[F_A(e)]\) for all \(e \in E\) and \((G, B) \subseteq f_{pu}(H, A)\). Hence \(y \in f_{pu}(F, A)\) and \((G, B) \subseteq f_{pu}(H, A)\) and \(f_{pu}(F, A) \triangle f_{pu}(H, A) = f_{pu}(F_A(e) \triangle G_A(e)) = f_{pu}[\emptyset] = \emptyset\) from Theorem 2.2. Since \(f_{pu}\) is irresolute open soft function. Then \(f_{pu}(F, A), f_{pu}(H, A)\) are semi open soft sets in \(Y\). Thus, \((Y, \tau_2, B)\) is also a soft semi regular space.
Theorem 4.5. Let \( (X, \tau_1, A) \) and \( (Y, \tau_2, B) \) be soft topological spaces and \( f_{pu} : SS(X)_A \rightarrow SS(Y)_B \) be soft function which is bijective, irresolute soft and irresolute open soft. If \( (X, \tau_1, A) \) is soft semi \( T_3 \)-space, then \( (Y, \tau_2, B) \) is also a soft semi \( T_3 \)-space.

Proof. Since \( (X, \tau_1, A) \) is soft semi \( T_3 \)-space, then \( (X, \tau_1, A) \) is soft semi regular \( T_1 \)-space. It follows that \( (Y, \tau_2, B) \) is also a soft semi \( T_1 \)-space from Theorem 4.2 and soft semi regular space from Theorem 4.4. Hence, \( (Y, \tau_2, B) \) is also a soft semi \( T_3 \)-space.

Theorem 4.6. Let \( (X, \tau_1, A) \) and \( (Y, \tau_2, B) \) be soft topological spaces and \( f_{pu} : SS(X)_A \rightarrow SS(Y)_B \) be soft function which is bijective, irresolute soft and irresolute open soft. If \( (X, \tau_1, A) \) is soft semi normal space, then \( (Y, \tau_2, B) \) is also a soft semi normal space.

Proof. Let \( (F, B) \), \( (G, B) \) be semi closed soft sets in \( Y \) such that \( (F, B) \cap (G, B) = \hat{\phi}_B \). Since \( f_{pu} \) is irresolute soft, then \( f_{pu}^{-1}(F, B) \) and \( f_{pu}^{-1}(G, B) \) are semi closed soft set in \( X \) such that \( f_{pu}^{-1}(F, B) \cap f_{pu}^{-1}(G, B) = f_{pu}^{-1}((F, B) \cap (G, B)) = f_{pu}^{-1}[(F, B) \cap (G, B)] = \hat{\phi}_A \) from Theorem 2.2. By hypothesis, there exist semi open soft sets \( (K, A) \) and \( (H, A) \) in \( X \) such that \( f_{pu}^{-1}(F, B) \subseteq (K, A) \), \( f_{pu}^{-1}(G, B) \subseteq (H, A) \) and \( (F, A) \cap (H, A) = \hat{\phi}_A \). It follows that \( (F, B) = f_{pu}[(F, B) \cap (G, B)] = f_{pu}[(F, B) \cap (G, B)] = f_{pu}[(G, B)] = f_{pu}[(G, B)] = f_{pu}[(H, A)] = \hat{\phi}_B \) from Theorem 2.2. Since \( f_{pu} \) is irresolute open soft function. Then \( f_{pu}(K, A), f_{pu}(H, A) \) are semi open soft sets in \( Y \). Thus, \( (Y, \tau_2, B) \) is also a soft semi normal space.

Corollary 4.1. Let \( (X, \tau_1, A) \) and \( (Y, \tau_2, B) \) be soft topological spaces and \( f_{pu} : SS(X)_A \rightarrow SS(Y)_B \) be soft function which is bijective, irresolute soft and irresolute open soft. If \( (X, \tau_1, A) \) is soft semi \( T_4 \)-space, then \( (Y, \tau_2, B) \) is also a soft semi \( T_4 \)-space.

Proof. It is obvious from Theorem 4.2 and Theorem 4.6.

5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [21] and easily applied to many problems having uncertainties from social life. In this paper, we introduce the notion of soft semi separation axioms.

In particular we study the properties of the soft semi regular spaces and soft semi normal spaces. We show that if \( x_E \) is semi closed soft set for all \( x \in X \) in a soft topological space \( (X, \tau, E) \), then \( (X, \tau, E) \) is soft semi \( T_1 \)-space. Also, we show that if a soft topological space \( (X, \tau, E) \) is soft semi \( T_3 \)-space, then \( \forall \; x \in X, \ x_E \) is semi closed soft set. Also, we show that the property of being semi \( T_i \)-spaces \( (i = 1, 2) \) is soft topological property under a bijection and irresolute open soft mapping. Further, the properties of being soft semi regular and soft semi normal are soft topological properties under a bijection, irresolute soft and irresolute open soft functions. Finally, we show that the property of being semi \( T_i \)-spaces \( (i = 1, 2, 3, 4) \)
is a hereditary property. We hope that the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical li

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