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On the gravity of center of sequence of fuzzy numbers

Zarife Zararsiz, Mehmet Şengönül

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Abstract. The aim of this work is to present the method for handling the definition of center of gravity for sequence of fuzzy numbers. We also investigate gravity of center of sequence of fuzzy numbers by using centroid defuzzification method and mean-max defuzzification method which are showed by z^*_{cdm} and z^*_{mm}, respectively. At the same time, we have tried to establish a relationship between limitation methods and mean-max defuzzification method.

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Corresponding Author: Mehmet Şengönül (msengonul@yahoo.com)

1. INTRODUCTION

In recent years, so many articles are gained to mathematical world about the sequence of fuzzy numbers. Almost all of these works, fuzzy sets which are provided by fuzzifying a certain value with respect to a proper membership function are used. The fuzzification is the process which converts a crisp value to fuzzy value. In the real world so many quantities that are seen as crisp and certain are not normally specific and certain. These quantities contain serious uncertainty.

If the form of uncertainty happens to arise imprecision, ambiguity, or vagueness, then the variable is probably fuzzy and can be represented by a membership function. Besides in the application a control command is given as a crisp value. Because of this it is needed to defuzzify the result of the fuzzy inference. A defuzzification is a process to get a non- fuzzy control action that best represents the possibility distribution of an inferred fuzzy control action. Defuzzification has the result of reducing a fuzzy set to a crisp set; of converting a fuzzy matrix to a crisp matrix; or of making a fuzzy number a crisp number. Furthermore the primary focus of the defuzzification method has been to explain the process of converting from fuzzy membership function to crisp version. Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion, [7]. The more detail about defuzzification can be found in [3].

In this paper by defuzzifying the sequence of fuzzy numbers with proper defuzzification method, we investigate some of the important and specific properties of sequence of fuzzy numbers which is called center of gravity of the sequence of the fuzzy numbers.

2. Preliminaries

A fuzzy number is a fuzzy subset of the $\mathbb{R}$ that is bounded, convex, normal and have a compact support, in other word a fuzzy number is characterized by a membership function $u : \mathbb{R} \to [0, 1]$ and satisfies the following properties:

`FN1.` u is normal, i.e., there exists an $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$,

`FN2.` u is fuzzy convex, i.e., for any $x, y \in \mathbb{R}$ and $\mu \in [0, 1]$,
$u[\mu x + (1 - \mu)y] \geq \min\{u(x), u(y)\}$,

`FN3.` u is upper semi-continuous,

`FN4.` The closure of $\{x \in \mathbb{R} : u(x) \geq 0\}$, denoted by u^0, is compact.

We will denote any triangular fuzzy number with $u = (u^-, \dot{u}, u^+)$ to construct our ideas which this notation was used in many papers, [8], [2]. We show the set of all fuzzy numbers by E^1 defined on the set $\mathbb{R}$ through all the text.

Let us denote the set of all sequences of fuzzy numbers by $w(E^2)$ that is

$$w(E^2) = \{u = (u_k) = ((u_k^-, \dot{u}_k, u_k^+)) : u : \mathbb{N} \to E^1, u(k) := (u_k^-, \dot{u}_k, u_k^+)\} \tag{2.1}$$

Here $u_k^-, \dot{u}_k, u_k^+$ represent first, middle and end points of general term of sequences of triangular fuzzy numbers, for every $k \in \mathbb{N}$, respectively. And degree of membership at $\dot{u}_k$ is

$$\begin{cases} 1, & \text{for the fuzzy numbers ,} \\ 0 \leq \varphi \leq 1, & \text{for the fuzzy sets} \end{cases}.$$

The real numbers $\dot{u}_k - u_k^-$ and $u_k^+ - \dot{u}_k$ are called the left, right indeterminateness of u_k, respectively. Among the some methods for defuzzification that have been proposed in the literature in recent years, are described here for defuzzifying fuzzy membership functions, [7].

Now we will list some famous defuzzification methods below:

First introduced defuzzification method is called centroid defuzzification method defined by the algebraic expression $z^*_{cdm} = \frac{\int \mu_C(z) z dz}{\int \mu_C(z) dz}$, where $\int$ denotes an algebraic integration, [8], [4]. An other defuzzification method is called weighted average method which defined by the algebraic expression
$z^*_{wam} = \frac{\sum \mu_C(\bar{z})\bar{z}}{\sum \mu_C(\bar{z})}$, here $\sum$ denotes the algebraic sum and $\bar{z}$ is the centroid of each symmetric membership function. The weighted average method is the most frequently used in fuzzy applications since it is one of the more computationally efficient methods. Unfortunately it is usually restricted to symmetrical output membership functions. The weighted average method is formed by weighting each membership function in the output by its respective maximum membership value. The third method is called mean-max method which given by the following equation

$z^*_{mm} = \frac{1}{n}\sum_{k=1}^{n} \max \mu_C(z_k)$, where n represents number of the fuzzy sets. The method is not restricted to symmetric membership functions.

The methods mentioned above are important defuzzification methods used in presently.

Defuzzification is necessary because, for example, we cannot instruct the voltage going into a machine to increase slightly, even if this instruction comes from a fuzzy controller we must alter its voltage by a specific amount. Defuzzification is a natural and necessary process. In fact, there is an analogous form of defuzzification in mathematics where we solve a complicated problem in the complex plane, find the real and imaginary parts of the solution, then decomplexify the imaginary solution back to the real numbers space [1]. There are numerous other methods for defuzzification that have not been presented here. A review of the literature will provide the details on some of these (see, for example, [2], [9]).

Let's denote the set of all real valued interval numbers by E_i. Any element of E_i is denoted by $\overline{x}$. That is $\overline{x} = \{x \in \mathbb{R} : x_l \leq x \leq x_r\}$.

The matrix defined by

$$c_{nk} = \begin{cases} \frac{1}{n+1} & , \quad 1 \leq k \leq n \\ 0 & , \quad \text{otherwise,} \end{cases} \qquad (\forall n, k \in \mathbb{N})$$

is called the *Cesàro matrix* and denoted by $C_1 = (c_{nk})$.

Let $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n, k \in \mathbb{N} = \{0, 1, 2, \ldots\}$. Then, we can say that A defines a matrix mapping if for every sequence $x = (x_k)$ the sequence $Ax = \{(Ax)_n\}$, the A-transform of x, exists where

$$(Ax)_n = \sum_k a_{nk} x_k, \quad (n \in \mathbb{N}). \tag{2.2}$$

The set of all interval numbers E_i is a metric space [6] with the metric d defined by

$$d(\overline{x}, \overline{y}) = \max\{|x_\ell - y_\ell|, |x_r - y_r|\}. \tag{2.3}$$

Moreover, it is known that E_i is a complete metric space. FN1, FN2, FN3 and FN4 imply that for each $\alpha \in [0, 1]$, the α-level set defined by $[u]^\alpha = \{x \in \mathbb{R} : u(x) \geq \alpha\}$ is in E_i, as well as the support u^0, i.e., $[u]^\alpha = [u_\ell(\alpha), u_r(\alpha)]$ for each $\alpha \in [0, 1]$.

Define a map $\overline{d} : E^1 \times E^1 \longrightarrow \mathbb{R}$ by $\overline{d}(u, v) = \sup_{0 \leq \alpha \leq 1} d([u]^\alpha, [v]^\alpha)$. It is known that E^1 is a complete metric space with the metric $\overline{d}$, [5].

Now we will give the new definitions about defuzzification method for the sequences of fuzzy sets. Furthermore we will define convergence of a sequence of fuzzy numbers without hold to $\alpha-$ cut sets.

Definition 2.1. A sequence $u = (u_k)$ of fuzzy numbers is said to be convergent to the fuzzy number u_0 if for each $\epsilon > 0$ there exists a positive integer n_0 such that $d(u_k, u_0) < \epsilon$ for all $k \geq n_0$, and we denote it by writing $\lim_k u_k = u_0$, where $d(u_k, u_0) = \sup_k \max\{|u_k^- - u_0^-|, |\dot{u}_k - \dot{u}_0|, |u_k^+ - u_0^+|\} < \epsilon$.

Let us denote convergent and null sequences spaces of fuzzy numbers by $c(E^1)$ and $c_0(E^1)$, respectively.

Definition 2.2. Let us suppose that $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ be a sequence of fuzzy numbers and $u_k^-, \dot{u}_k, u_k^+$ are increasing sequence of the real numbers. Then the gravity of center of the sequence (u_k) is defined by

$$z^*_{cdm} = \lim_n \frac{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)xdx + \sum_{k=1}^n [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)xdx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)xdx]}{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)dx + \sum_{k=1}^n [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)dx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)dx]}, \tag{2.4}$$

where α_k is obtained from common solutions of the functions $u_k^+(x)$ and $u_{k+1}^-(x)$.

Similarly, if $u_k^-, \dot{u}_k, u_k^+$ are decreasing sequence of the real numbers, then the gravity of center of the sequence $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ is defined as follows:

$$z^*_{cdm} = \lim_n \frac{\int_{\dot{u}_1}^{u_1^+} u_1^+(x)xdx + \sum_{k=1}^n [\int_{\nu_k}^{\dot{u}_k} u_k^-(x)xdx + \int_{\dot{u}_k}^{\nu_k} u_{k+1}^+(x)xdx]}{\int_{\dot{u}_1}^{u_1^+} u_1^+(x)dx + \sum_{k=1}^n [\int_{\nu_k}^{\dot{u}_k} u_k^-(x)dx + \int_{\dot{u}_k}^{\nu_k} u_{k+1}^+(x)dx]} \tag{2.5}$$

where ν_k is obtained from common solutions of the functions $u_{k+1}^+(x)$ and $u_k^-(x)$.

In [8], centroid defuzzification method is given for finite number of fuzzy sets. Similar to this definition, we have given a centroid defuzzification method for sequence of fuzzy numbers as mentioned above. So we generalized the definition of Sugeno, [8].

Definition 2.3. Let us suppose that $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ be a sequence of fuzzy numbers. Then mean-max defuzzification method for the sequence of fuzzy numbers (u_k) is defined by

$$z^*_{mm} = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^n \max \mu_C(\dot{u}_k). \tag{2.6}$$

3. Major Section

Let's consider the set $w(E^*) = \{u = (u_k) \in w(E^2) : \{u_k : k \in \mathbb{N}\} \text{is a finite set}\}$.

Theorem 3.1. *If $(u_k) = ((u_k^-, \dot{u}_k, u_k^+)) \in w(E^*)$ then z^*_{cdm} always exists for the sequence (u_k).*

Proof. Let us suppose that $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ has finite non zero elements and the sequence $(\dot{u}_k)$ be increasing sequence of the real numbers. Then

$$\begin{aligned} z^*_{cdm} &= \frac{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)xdx + \sum_{k=1}^\infty [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)xdx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)xdx]}{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)dx + \sum_{k=1}^\infty [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)dx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)dx]} \\ &= \frac{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)xdx + \sum_{k=1}^n [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)xdx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)xdx]}{\int_{u_1^-}^{\dot{u}_1} u_1^-(x)dx + \sum_{k=1}^n [\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)dx + \int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)dx]} \\ &\quad \frac{+ \sum_{k\geq n+1} [\int_{\dot{u}_k}^{\alpha_k} 0dx + \int_{\alpha_k}^{\dot{u}_{k+1}} 0dx]}{+ \sum_{k\geq n+1} [\int_{\dot{u}_k}^{\alpha_k} 0dx + \int_{\alpha_k}^{\dot{u}_{k+1}} 0dx]}. \end{aligned} \tag{3.1}$$

Since the integrals $\int_{u_1^-}^{\dot{u}_1} u_1^-(x)xdx$, $\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)xdx$, $\int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)xdx$,

$\int_{u_1^-}^{\dot{u}_1} u_1^-(x)dx$, $\int_{\dot{u}_k}^{\alpha_k} u_k^+(x)dx$ and $\int_{\alpha_k}^{\dot{u}_{k+1}} u_{k+1}^-(x)dx$ always exist in the sense of the Riemann, this shows that the fraction, (3.1) exists, in other words the z^*_{cdm} exists.

Similar to this process, if the sequence $(\dot{u}_k)$ is decreasing sequence of the real numbers then again we can see that z^*_{cdm} exists. □

Now we will give some applications of the Theorem 3.1.

Example 3.2. Let's take a sequence of fuzzy numbers in $w(E^*)$ and the general term of the sequence is given as in the following;

$$\tilde{u}_k = \begin{cases} \left(\frac{4k-4}{k}, \frac{5k-4}{k}, \frac{6k-4}{k}\right), & k \leq 2 \\ 0, & k > 2 \end{cases}$$

So by using the centroid defuzzification method we determine the center of gravity of the sequence of the fuzzy numbers (u_k) as $z^*_{cdm} = 1.653$.

This example shows that the gravity of center z^*_{cdm} of the sequence $u = (u_k)$ of fuzzy numbers may be nonzero even if $u = (u_k) \in c_0(E^2)$.

Example 3.3. Let us consider the sequence

$$(u_k) = ((u_k^-, \dot{u}_k, u_k^+)) = ((\frac{1}{(k+2)^2}, \frac{1}{(k+1)^2}, \frac{1}{k^2}))$$

of fuzzy numbers. The membership function of the sequence (u_k) of fuzzy numbers can be given as follows:

$$u_k(x) = \begin{cases} u_k^-(x), & if \quad x \in [\frac{1}{(k+2)^2}, \frac{1}{(k+1)^2}] \\ u_k^+(x), & if \quad x \in [\frac{1}{(k+1)^2}, \frac{1}{k^2}] \\ 0, & other \;\; wise \end{cases}$$

$$= \begin{cases} \frac{(x(k+2)^2-1)(k+1)^2k^2}{(k+2)^2(-2k-1)}, & if \quad x \in [\frac{1}{(k+2)^2}, \frac{1}{(k+1)^2}] \\ \frac{(k+1)^2}{2k+1}[1-k^2x], & if \quad x \in [\frac{1}{(k+1)^2}, \frac{1}{k^2}] \\ 0, & other \;\; wise \end{cases}$$

From common solutions of the $u_k^-(x)$ and $u_{k+1}^+(x)$ we have the sequence $\alpha_k = \frac{16+64k+85k^2+48k^3+10k^4}{2(2+3k+k^2)^2(2+6k+3k^2)}$. Thus, from (2.4), we can write, using with a software programme,

$$z^* = \lim_n \frac{\int_{\frac{1}{4}}^{1} u_1^+(x)xdx + \sum_{k=1}^n [\int_{\alpha_k}^{\frac{1}{(k+1)^2}} u_k^-(x)xdx + \int_{\frac{1}{(k+2)^2}}^{\alpha_k} u_{k+1}^+(x)xdx]}{\int_{\frac{1}{4}}^{1} u_1^+(x)dx + \sum_{k=1}^n [\int_{\alpha_k}^{\frac{1}{(k+1)^2}} u_k^-(x)dx + \int_{\frac{1}{(k+2)^2}}^{\alpha_k} u_{k+1}^+(x)dx]}$$
$$= 0.678488,$$

that is the sequence of fuzzy numbers

$$(u_k) = ((u_k^-, \dot{u}_k, u_k^+)) = ((\frac{1}{(k+2)^2}, \frac{1}{(k+1)^2}, \frac{1}{k^2}))$$

has a gravity of center and this center is 0.678488.

Example 3.4. Similar to Example 3.3, let consider the sequence

$$(u_k) = ((u_k^-, \dot{u}_k, u_k^+)) = ((\frac{1}{k+2}, \frac{1}{k+1}, \frac{1}{k})).$$

The membership function of the sequence (u_k) of fuzzy numbers can be given as follows:

$$u_k(x) = \begin{cases} u_k^-(x), & if \quad x \in [\frac{1}{k+2}, \frac{1}{k+1}] \\ u_k^+(x), & if \quad x \in [\frac{1}{k+1}, \frac{1}{k}] \\ 0, & other \;\; wise \end{cases}$$
$$= \begin{cases} (k+1)(k+2)x - (k+1), & if \quad x \in [\frac{1}{k+2}, \frac{1}{k+1}] \\ (k+1) - k(k+1)x, & if \quad x \in [\frac{1}{k+1}, \frac{1}{k}] \\ 0, & other \;\; wise \end{cases}$$

From common solutions of the $u_k^-(x)$ and $u_{k+1}^+(x)$ we have the sequence $\alpha_k = \frac{2k+3}{2(k+2)(k+1)}$. Thus, from (2.4), we can write

$$z^*_{cdm} = \lim_n \frac{\int_{\frac{1}{2}}^1 u_1^+(x)xdx + \sum_{k=1}^n [\int_{\alpha_k}^{\frac{1}{k+1}} u_k^-(x)xdx + \int_{\frac{1}{k+2}}^{\alpha_k} u_{k+1}^+(x)dx]}{\int_{\frac{1}{2}}^1 u_1^+(x)dx + \sum_{k=1}^n [\int_{\alpha_k}^{\frac{1}{k+1}} u_k^-(x)dx + \int_{\frac{1}{k+2}}^{\alpha_k} u_{k+1}^+(x)dx]}$$
$$=0.416667$$

this means that the gravity point of sequence of fuzzy numbers $(u_k) = ((u_k^-, \dot{u}_k, u_k^+)) = ((\frac{1}{k+2}, \frac{1}{k+1}, \frac{1}{k}))$ is 0.416667.

Theorem 3.5. *Suppose that the sequence $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ be convergent to fuzzy number $(u_0^-, \dot{u}_0, u_0^+)$. Generally, the gravity of center of the sequence $(u_k) = ((u_k^-, \dot{u}_k, u_k^+))$ is not equal to gravity of center of $(u_0^-, \dot{u}_0, u_0^+)$.*

Proof. If we consider Example 3.2 then we see that the sequence

$$\tilde{u}_k = \begin{cases} \left(\frac{4k-4}{k}, \frac{5k-4}{k}, \frac{6k-4}{k}\right), & k \leq 2 \\ 0, & k > 2 \end{cases}$$

is convergent to fuzzy zero. But gravity of center of it is $z^*_{cdm} = 1.653$. □

Let us consider the sequence of fuzzy numbers

$$(u_k) = \begin{cases} (-2, -1, 0), & if \;\; k \;\; is \;\; odd \\ (0, 1, 2), & if \;\; k \;\; is \;\; even \end{cases}. \tag{3.2}$$

The sequence (u_k) is not a convergent sequence of fuzzy numbers. Even so, from (2.4), z^*_{cdm} the gravity of center of (u_k) is determined as follows;

$$z^*_{cdm} = \lim_n \frac{\int_{-2}^{-1}(x^2+2x)dx + \sum_{k=1}^n [\int_{-1}^0 -x^2dx + \int_0^1 x^2dx] + \int_1^2 (2x - x^2)dx}{\int_{-2}^{-1}(x+2)dx + \sum_{k=1}^n [\int_{-1}^0 -xdx + \int_0^1 xdx] + \int_1^2 (2-x)dx}$$
$$= 0. \tag{3.3}$$

This shows that any sequence of fuzzy numbers may have a gravity of center which need not convergent to a fuzzy number.

Now, we can give a proposition as follows:

Proposition 3.6. *Let* (u_k) *be any element of the set* $w(E^2) \setminus w(E^*)$ *. Then the real value* z^*_{cdm} *is equal to the limit of the sequence* (r_n) *of real numbers, where the sequence* (r_n) *which obtain with defuzzification term by term of the sequence* (u_k)*.*

If we consider Definition 2.3, in fact that, the real value z^*_{mm} is equal to the C_1-transform of the sequence $\max \mu(\dot{u}_k)$, for every $k \in \mathbb{N}$. Therefore, if we want to find precise value for any sequence fuzzy numbers with mean-max defuzzification method then it is sufficient to find C_1-transform of the sequence of real numbers $(\dot{u}_k)$, where $\dot{u}_k$ have been considered to have the maximum membership value.

Now we can give a theorem as follows:

Theorem 3.7. *Let* (u_k) *be any sequence of fuzzy numbers in* $\ell_\infty(E^2) \setminus c(E^2)$*. Then gravity of center with the mean-max defuzzification method of the sequence* (u_k) *is equal to* C_1-*transform of the sequence of real numbers* (r_k)*, where* $r_k = \max \mu(\dot{u}_k)$*.*

Proof. Let suppose that $(u_k) \in \ell_\infty(E^2) \setminus c(E^2)$. For all $x \in \mathbb{R}$, if $\max \mu_{u_k}(x) = r_k$ then $\lim_n \frac{1}{n} \sum_{k=1}^{n} \max \mu(\dot{u}_k) = \lim_n \frac{1}{n} \sum_{k=1}^{n} r_k = (C_1 r)_n$. If we take the sequence (u_k) as in Example 3.4, the gravity of center of (u_k) is equal to 0. This result, as geometrical, is meaningless so we should chose the sequence (u_k) in $\ell_\infty(E^2) \setminus c(E^2)$. □

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Zarife Zararsiz (zarifezararsiz@nevsehir.edu.tr)
Nevşehir University, Science and Art Faculty, Department of Mathematics, 50300, Nevşehir, Turkey

Mehmet Şengönül (msengonul@yahoo.com)
Nevşehir University, Science and Art Faculty, Department of Mathematics, 50300, Nevşehir, Turkey